Algebra Handout

Denver Math Club

August 2018

1. Evaluate $1^3 + 2^2 + 3^3 + 4^2 + 5^3 + 6^2 + \ldots + 19^3 + 20^2$.

2. A 5 L sample of Solution A is made of 5% sugar and 8% salt. Solution B is made of 18% sugar and 2% salt. How many mL of Solution B must be added to Solution A to ensure that the sugar and salt contents of the resulting solution are equal? Express your answer as a decimal to the nearest tenth.

3. A giant pencil costs 12 times as much as a pencil. A pencil costs 3 times as much as an eraser. At the very minimum, how many pencils are needed to exceed the combined cost of a giant pencil and an eraser?

4. A car traveling between two cities moves at 65 mph for half the distance, 45 mph for the next quarter and 35 mph for the remaining quarter. In mph, what is the average speed of the car? Express your answer as a common fraction.

5. What is the sum of the solutions to the equation $\sqrt[4]{x} = \dfrac{12}{7 - \sqrt[4]{x}}$?

6. In an arithmetic sequence, the first, third and sixth terms form a geometric sequence. Compute the common ratio of this geometric sequence, expressing your answer as a common fraction.

7. Three taps, A, B and C, are being used to fill a swimming pool. Tap A can fill the pool in 6 hours. If only tap B is turned on until half of the swimming pool is filled, and then only tap C is turned on until the remainder of the pool is filled, 8 hours will have elapsed. In minutes, what is the maximum time it will take to fill the pool if taps A, B and C are all turned on?

8. Let $S(r)$ denote the sum of the geometric series $12 + 12r + 12r^2 + 12r^3 + \ldots$. Let a satisfy $S(a)S(-a) = 2016$. Find $S(a) + S(-a)$.

9. The expression $(x - y + 2z)^{10}$ is expanded. Compute the sum of the coefficients of all terms for which the sum of the exponents of the x, y and z is even.

10. Let $S = \frac{1}{1\cdot2} + \frac{1}{3\cdot4} + \ldots + \frac{1}{99\cdot100}$ and $T = \frac{1}{51\cdot100} + \frac{1}{52\cdot99} + \ldots + \frac{1}{99\cdot52} + \frac{1}{100\cdot51}$. Compute $\frac{S}{T}$, expressing your answer as a common fraction.