Algebra

1. Find all solutions in integers to the equation mn = 2m + 7n + 22.

2. Find the sum of the reciprocals of the roots of the polynomial x^3 + 5x - 7 = 0.

3. Find the sum of the cubes of the reciprocals of the roots of the polynomial x^3 + 5x - 7 = 0.

4. Compute $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{2021}{2022!}$.

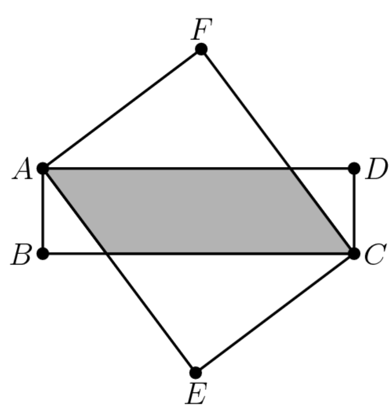
5. Compute the positive difference between the two real solutions to the equation $(x-1)(x-4)(x-2)(x-8)(x-5)(x-7)+48\sqrt{3}=0.$

Geometry

1. Find the area of triangle ABC if angle ABC is 60 degrees, angle BCA is 45 degrees, and side AB has length 1.

2. Triangle ABC has AB = 13, BC = 14, CA = 15.
a) What is the area of ABC?
b) The angle bisector of angle A intersects side BC at D. What is BD?
c) E lies on AB such that AE = 4 and F lies on AC such that AF = 5. What is the area of AEF?

3. In the diagram below, $ABCD$ is a rectangle with side lengths $AB=3$ and $BC=11$, and $AECF$ is a rectangle with side lengths $AF=7$ and $FC=9$, as shown. What is the area of the shaded region?



Combinatorics

1. How many ways are there to put 4 balls into 3 boxes if
   a) Both the balls and the boxes are distinguishable
   b) The balls are distinguishable but the boxes are not
   c) The boxes are distinguishable but the balls are not
   d) Neither the balls nor the boxes are distinguishable

2. How many arrangements of the word DENVER are there?

3. How many arrangements of the word DENVER are there that don't have the Es next to each other?

4. You have two boxes, Box 1 and Box 2. Box 1 contains 2 red balls and 3 blue balls, while Box 2 contains 3 red balls and 3 blue balls. You select a box and draw a ball randomly. Given that you drew a blue ball, find the probability that you chose box 1.

5. Prair rolls a standard six-sided die four times, and notices that the rolls form a strictly increasing sequence, where each roll is strictly greater than the immediately preceding roll. What is the probability that the second number she rolled was a 3?

6. A special deck of cards contains 49 cards, each labeled with a number from 1 to 7 and colored with one of seven colors. Each number-color combination appears on exactly one card. Sharon will select a set of eight cards from the deck at random. Given that she gets at least one card of each color and at least one card with each number, what is the probability that Sharon can discard one of her cards and *still* have at least one card of each color and at least one card with each number?

<u>Number Theory</u>

1. Find a) the number of divisors, b) the sum of the divisors, and c) the product of the divisors of 2023.

2. Find the remainder when 8^(2023) is divided by a) 9, b) 11, and c) 15.

3. Find all positive integers n such that n^2 + 85n + 2017 is a perfect square.

4. Find integers x and y such that 37x + 201y = 1.

5. For any positive integer a, $\sigma(a)$ denotes the sum of the positive integer divisors of a. Let n be the least positive integer such that $\sigma(a^n) - 1$ is divisible by 2021 for all positive integers a. Find the sum of the prime factors in the prime factorization of n.