Denver Math Club<br>November Meeting<br>Combinatorics

1. How many ways are there to rearrange the letters in the word BANANA?
2. Each edge of a cube is colored either red or black. Every face of the cube has at least one black edge. What is the smallest number possible of black edges?
3. Henry's Hamburger Haven offers its hamburgers with the following condiments: ketchup, mustard, mayonnaise, tomato, lettuce, pickles, cheese, and onions. A customer can choose one, two, or three meat patties and any collection of condiments. How many different kinds of hamburgers can be ordered?
4. Using the letters $A, M, O, S$, and $U$, we can form five-letter "words". If these "words" are arranged in alphabetical order, which position does the "word" USAMO occupy?
5. Coin $A$ is flipped three times and coin $B$ is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?
6. Two counterfeit coins of equal weight are mixed with 8 identical genuine coins. The weight of each of the counterfeit coins is different from the weight of each of the genuine coins. A pair of coins is selected at random without replacement from the 10 coins. A second pair is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 of the selected coins are genuine?
7. One commercially available ten-button lock may be opened by pressing -- in any order -- the correct five buttons. Suppose that these locks are redesigned so that sets of as many as nine buttons or as few as one button could serve as combinations. How many additional combinations would this allow?
8. When a certain biased coin is flipped five times, the probability of getting heads exactly once is not equal to 0 and is the same as that of getting heads exactly twice. Let $i / j$, in lowest terms, be the probability that the coin comes up heads in exactly 3 out of 5 flips. Find $\mathrm{i}+\mathrm{j}$.
9. Every card in a deck has a picture of one shape - circle, square, or triangle, which is painted in one of the three colors - red, blue, or green. Furthermore, each color is applied in one of three shades - light, medium, or dark. The deck has 27 cards, with every shape-color-shade combination represented. A set of three cards from the deck is called complementary if all of the following statements are true:
i. Either each of the three cards has a different shape or all three of the card have the same shape.
ii. Either each of the three cards has a different color or all three of the cards have the same color.
iii. Either each of the three cards has a different shade or all three of the cards have the same shade.

How many different complementary three-card sets are there?
10. In a shooting match, eight clay targets are arranged in two hanging columns of three targets each and one column of two targets. A marksman is to break all the targets according to the following rules:

1) The marksman first chooses a column from which a target is to be broken.
2) The marksman must then break the lowest remaining target in the chosen column.

If the rules are followed, in how many different orders can the eight targets be broken?

11. Ninety-four bricks, each measuring $4 " \mathrm{X} 10 " \mathrm{X} 19 "$ are to be stacked one on top of another to form a tower 94 bricks tall. Each brick can be oriented so it contributes 4 " or 10 " or 19 " to the total height of the tower. How many different tower heights can be achieved using all ninety-four of the bricks?
12. Define a domino to be an ordered pair of distinct positive integers. A proper sequence of dominos is a list of distinct dominos in which the first coordinate of each pair after the first equals the second coordinate of the immediately preceding pair, and in which ( $\mathrm{i}, \mathrm{j}$ ) and ( j , i)do not both appear for any $i$ and $j$. Let $D_{40}$ be the set of all dominos whose coordinates are no larger than 40 . Find the length of the longest proper sequence of dominos that can be formed using the dominos of $D_{40}$.

