

Denver Math Club
ARML Practice Individual Problems

1. Given that a , b , and c are positive integers such that $a \cdot b \cdot c$ is a multiple of 2016, compute the least possible value of $a + b + c$.

2. Compute
$$\frac{\lfloor \sqrt[4]{1} \rfloor \cdot \lfloor \sqrt[4]{3} \rfloor \cdot \lfloor \sqrt[4]{5} \rfloor \cdots \lfloor \sqrt[4]{2015} \rfloor}{\lfloor \sqrt[4]{2} \rfloor \cdot \lfloor \sqrt[4]{4} \rfloor \cdot \lfloor \sqrt[4]{6} \rfloor \cdots \lfloor \sqrt[4]{2016} \rfloor}.$$

3. Compute the number of permutations x_1, \dots, x_6 of the integers $1, \dots, 6$ such that $x_{i+1} \leq 2x_i$ for all $i, 1 \leq i < 6$.
4. Compute the least possible non-zero value of $A^2 + B^2 + C^2$ such that A , B , and C are integers satisfying $A \log 16 + B \log 18 + C \log 24 = 0$.

5. Compute the greatest integer x such that
$$\left\lfloor \sqrt{\left\lfloor \sqrt{\left\lfloor \sqrt{x} \right\rfloor} \right\rfloor} \right\rfloor = 2.$$

6. Compute the number of ordered pairs of integers (x, y) such that
$$\frac{1}{x} + \frac{540}{xy} = 2.$$
7. A positive integer has the Kelly Property if it contains a zero in its base-17 representation. Compute the number of positive integers less than 1000 (base 10) that have the Kelly Property.

8. An urn contains 4 green balls and 6 blue balls. A second urn contains 16 green balls and N blue balls. A single ball is drawn at random from each urn. The probability that both balls are of the same color is 0.58. Find N .
9. The AIME Triathlon consists of a half-mile swim, a 30-mile bicycle ride, and an eight-mile run. Tom swims, bicycles, and runs at constant rates. He runs five times as fast as he swims, and he bicycles twice as fast as he runs. Tom completes the AIME Triathlon in four and a quarter hours. How many minutes does he spend bicycling?
10. Find the number of five-digit positive integers, n , that satisfy the following conditions:
- (a) the number n is divisible by 5,
 - (b) the first and last digits of n are equal, and
 - (c) the sum of the digits of n is divisible by 5.