1. Soda is sold in packs of 6, 12 and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

2. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed so that 75% of the remaining balls are red?

3. Suppose r, s, and t are nonzero reals such that the polynomial $x^2 + rx + s$ has s and t as roots, and the polynomial $x^2 + tx + r$ has 5 as a root. Compute s.

4. Compute the sum of all two-digit positive integers x such that for all three-digit (base 10) positive integers $\underline{abc}$, if $\underline{abc}$ is a multiple of x, then the three-digit (base 10) number $\underline{bca}$ is also a multiple of x.

5. Let $A = (2^2-1)(3^2-1)(4^2-1)...(2024^2-1)$ and $B = (2^2)(3^2)(4^2)...(2024^2)$. Compute A/B, expressed as a common fraction.

6. Compute the sum of all positive integers n for which the expression $\frac{n+7}{\sqrt{n-1}}$ is an integer.

7. For some real number c, the graphs of the equation $y = |x-20| + |x+18|$ and the line $y = x + c$ intersect at exactly one point. What is c?

8. Compute the number of positive integers that divide at least two of the integers in the set $\{1^1, 2^2, 3^3, 4^4, 5^5, 6^6, 7^7, 8^8, 9^9, 10^{10}\}$.

9. Compute the sum of all positive integers $50 \leq n \leq 100$ such that $2n + 3 \nmid 2^{n!} - 1$.

10. Let f(x) be a quotient of two quadratic polynomials. Given that $f(n) = n^3$ for all $n \in \{1, 2, 3, 4, 5\}$, compute f(0). Express your answer as a common fraction.

Answers:

1. 5
2. 100
3. 29 or -6
4. 64
5. 2025/4048
6. 89
7. 18
8. 22
9. 222
10. 24/17