

Algebra Handout

1. A bucket of water is already $\frac{3}{4}$ full, and adding 100 mL of water will make it $\frac{13}{16}$ full. How much water is in a completely filled bucket?
2. Lisa squares the two-digit positive integers AC and BC . If the two results are 2340 apart, what is $A + B + C$?
3. Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?
4. Two freight trains of lengths a and b miles are travelling at 100 and 150 mph on parallel tracks. If the two trains take the same amount of time to completely travel through a 5 mile tunnel, what is the ratio $a : b$?
5. What is the sum of the roots of $x^3 - 3x^2 + 3x - 7 = 0$?
6. The monic polynomial $p(x)$ has degree 10 and roots $-5, -4, -3, -2, 1, 2, 3, 4, 5$. Find the largest possible value of $p(0)$.
7. If $\frac{1}{a} + \frac{1}{b} = 1$, $a^2 + b^2 = 8$, and $b < 0$, what is $a^3 + b^3$?
8. Let x_1, x_2, \dots, x_{10} be 10 numbers. Suppose that $x_i + 2x_{i+1} = 1$ for each i from 1 through 9. What is the value of $x_1 + 512x_{10}$?
9. Simplify $\sqrt[3]{8 + 3\sqrt{21}} + \sqrt[3]{8 - 3\sqrt{21}}$.
10. Find the number of positive integers n less than 1000 for which there exists a positive real number x such that $n = x \lfloor x \rfloor$.
11. Compute the sum of the coefficients in the polynomial $(5w + 4x + 3y + 2z)^3$.
12. Suppose that x, y , and z are three positive numbers that satisfy the equations $xyz = 1, x + \frac{1}{z} = 5, y + \frac{1}{x} = 29$. Compute $z + \frac{1}{y}$.
13. What is $\frac{1}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{5}{32} + \frac{8}{64} + \frac{13}{128} + \dots$, where the numerators are the Fibonacci numbers?