Denver Math Club February Meet State Sprint Round

1. Seven consecutive integers sum to 238. What is the greatest of these seven integers?

2. What is the least natural number that can be added to 51,706 to form a palindrome?

3. Annalise's bedroom has floor dimensions 14 feet by 16 feet, with height 10 feet. If one bucket of green paint can cover 100 square feet, how many buckets of green paint must she buy to completely cover the walls and the ceiling of her bedroom?

4. Let β be the relation defined by A β B = 4A - B. What is the value of 7 β (5 β 8)?

5. The mean of five positive integers is 1.5 times their median. Four of the integers are 8, 18, 36 and 62, and the largest integer is not 62. What is the largest integer?

6. If three standard, six-faced dice are rolled, what is the probability that the sum of the three numbers rolled is 9? Express your answer as a common fraction.

7. The integers 2, 3, 4, 6, and 9 will be put in random order to make a positive five-digit integer. What is the probability that the resulting integer will be divisible by 12? Express your answer as a common fraction.

8. If $12_3 + 12_5 + 12_7 + 12_9 + 12_x = 101110_2$, what is the value of x?

9. If the point (x, x_{2}) is equidistant from (-2, 5) and (3, -2), what is the value of x?

10. In some languages, every consonant must be followed by a vowel. How many seven-level "words" can be made from the Hawaiian word MAKAALA if each consonant must be followed by a vowel?

11. Given that -4 is a solution to $x^2+bx-36=0$, what is the value of b?

12. What is the units digit of 2002^{2002} ?

13. Jane has seven identical yellow socks, three identical red socks, four identical green socks and two identical orange socks in her drawer. If she randomly selects two socks from her drawer, what is the probability that they will be the same color? Express your answer as a common fraction.

14. A right triangle has sides with lengths 8 cm, 15 cm and 17 cm. A circle is inscribed in the triangle. In centimeters, what is the radius of the circle?

15. The student council at Round Junior High School has eight members who meet at a circular table. If the four officers must sit together in any order, how many distinguishable circular seating orders are possible? Two seating orders are distinguishable if one is not a rotation of the other.

16. In isosceles trapezoid ABCD, shown here, AB = 4 units and CD = 10 units. Points E and F are on CD with BE parallel to AD and AF parallel to BC. AF and BE intersect at point G. What is the ratio of the area of triangle EFG to the area of trapezoid ABCD? Express your answer as a common fraction.



17. There are twelve different mixed numbers that can be created by substituting three of the numbers 1, 2, 3 and 5 for a, b and c in the expression $a\frac{b}{c}$, where b < c. What is the mean of these twelve mixed numbers? Express your answer as a mixed number.

18. The function $f(n) = a \cdot n! + b$, where a and b are positive integers, is defined for all positive integers. If the range of f contains two numbers that differ by 20, what is the least possible value of f (1)?

19. For how many two-element subsets $\{a, b\}$ of the set $\{1, 2, 3, ..., 36\}$ is the product ab a perfect square?

20. In three flips of an unfair coin the probability of getting three heads is the same as the probability of getting exactly two tails. What is the ratio of the probability of flipping a tail to the probability of flipping a head? Express your answer as a common fraction in simplest radical form.

21. A bag contains red balls and white balls. If five balls are to be pulled from the bag, with replacement, the probability of getting exactly three red balls is 32 times the probability of getting exactly one red ball. What percent of the balls originally in the bag are red?

22. How many diagonals does a convex nonagon have?

23.What is the arithmetic mean (average) of the areas of all non-congruent rectangles with integer side lengths and perimeter 8 units? Express your answer as a decimal to the nearest tenth.

24. How many ordered triples of primes exist for which the sum of the members of the triple is 24?

25.In square units, what is the largest possible area a rectangle inscribed in the triangle shown here can have?

10 17 21