1. The area of trapezoid $A B C D$ is $164 \mathrm{~cm}^{2}$. The altitude is $8 \mathrm{~cm}, A B$ is 10 cm , and $C D$ is 17 cm . What is $B C$, in centimeters?

2. Three circular arcs of radius 5 units bound the region shown. Arcs $A B$ and $A D$ are quarter-circles, and arc $B C D$ is a semicircle. What is the area, in square units, of the region?

3. A circle of radius 1 has four circles $\omega_{1}, \omega_{2}, \omega_{3}$, and $\omega_{4}$ of equal radius internally tangent to it, so that $\omega_{1}$ is tangent to $\omega_{2}$, which is tangent to $\omega_{3}$, which is tangent to $\omega_{4}$, which is tangent to $\omega_{1}$, as shown. The radius of the circle externally tangent to
$\omega_{1}, \omega_{2}, \omega_{3}$, and $\omega_{4}$ has radius r . Find r , expressed in simplest radical form.

4. In rectangle ABCD , let E lie on CD , and let F be the intersection of AC and BE . If the area of triangle ABF is 45 and the area of triangle CEF is 20 , find the area of quadrilateral ADEF .
5. Find the area of the largest square that can be inscribed in a regular hexagon of side length 1 . Express your answer in simplest radical form.
6. Rectangle ABCD has $\mathrm{AB}=24$ and $\mathrm{BC}=7$. Let d be the distance between the centers of the incircles of triangle ABC and triangle CDA. Find $\mathrm{d}^{2}$.
7. Hexagon ABCDEF has an inscribed circle $\Omega$ that is tangent to each of its sides. If $\mathrm{AB}=12, \angle \mathrm{FAB}=120$ degrees, and $<\mathrm{ABC}=150$ degrees, find the radius of $\Omega$. Express your answer in simplest radical form.
8. Construct triangles ABC and DEF such that $\mathrm{AB}=10, \mathrm{BC}=11, \mathrm{CA}=12, \mathrm{C}$ lies on segment $A D$, $A$ lies on segment $B E$, $B$ lies on segment $C F$, and $C D=A E=B F=1$. Find the ratio of the area of triangle DEF to the area of triangle ABC , expressing your answer as a common fraction.
9. Rectangle HOMF has $\mathrm{HO}=11$ and $\mathrm{OM}=5$. Triangle ABC has orthocenter H and circumcenter $\mathrm{O} . \mathrm{M}$ is the midpoint of BC and altitude AF meets BC at F . Find the length of BC.
10. All of the faces of the convex polyhedron $P$ are congruent isosceles (but NOT equilateral) triangles that meet in such a way that each vertex of the polyhedron is the meeting point of either ten base angles of the faces or three vertex angles of the faces. (An isosceles triangle has two base angles and one vertex angle.) Find the sum of the numbers of faces, edges, and vertices of $P$.

Answers:
1.10
2. 50
3. 3-2sqrt2
4. 55
5. 12-6sqrt(3)
6. 325
7. $9+3$ sqrt 3
8. 343/264
9. 28
10. 182

