## Denver Math Club

January 2022 Meeting
Mathcounts State Sprint

1. Margaret picks a random letter from the word BANANA. What is the probability she does not pick the B?
2. If $f(x)=3 x+1$, what is $f(f(f(2)))$ ?
3. In the figure shown, the ratio of BD to DC is 4 to 3 . The area of $\triangle A B D$ is 24 square centimeters. What is the area of $\triangle A D C$ ?

4. What is the largest possible product when three different numbers from the set $\{-3,-2,-1,4,5\}$ are multiplied?
5. Jose is 4 years younger than Zack. Zack is 3 years older than Inez. Inez is 15 years old. How old is Jose?
6. What time and day was it 2022 minutes after midnight on January 1,2022 ?
7. Three congruent circles with centers $P, Q$, and $R$ are tangent to the sides of rectangle $A B C D$ as shown. The circle centered at Q has diameter 4 and passes through points P and R . What is the area of the rectangle?

8. How many trailing zeros are there in the base-ten expansion of 32 !?
9. In a recent basketball game, Shenille attempted only three-point shots and two-point shots. She was successful on $20 \%$ of her three-point shots and $30 \%$ of two-point shots. Shenille attempted 30 shots. How many points did she score?
10. A circle with radius 1 is inscribed in a square and circumscribed about another square. Find the ratio of the circle's shaded area to the area between the two squares.

11. If n is an integer, such that $2 \leq n \leq 2010$, for how many values of n is $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right) \ldots\left(1+\frac{1}{n}\right)$ equal to a positive integer?
12. A quadrilateral has vertices at $(1,2),(2,1),(7,6)$, and $(8,9)$. What is its area?
13. A student council must select a two-person welcoming committee and a three-person planning committee from among its members. There are exactly 10 ways to select a two-person team for the welcoming committee. It is possible for students to serve on both committees. In how many different ways can a three-person planning committee be selected?
14. The area of each of the four congruent L -shaped regions of this 100 -inch by 100 -inch square is $3 / 16$ of the total area. How many inches long is the side of the center square?

15. A pair of six-sided dice are labeled so that one die has only even numbers (two each of 2,4 , and 6 ), and the other die has only odd numbers (two each of 1,3 , and 5). The pair of dice is rolled. What is the probability that the sum of the numbers on the tops of the two dice is 7 ?
16. Suppose f is a quadratic function defined by $f(x)=a x^{2}+b x+c$ for some numbers $\mathrm{a}, \mathrm{b}$, and c. If $g(x)=x-2$ and $f(g(x))=2 x^{2}-5 x+19$ for some values of x , what is the value of $\mathrm{a}+\mathrm{b}+\mathrm{c}$ ?
17. A sphere is inscribed in a right cone with base radius 12 cm and height 24 cm , as shown. The radius of the sphere can be expressed as $a \sqrt{c}-a \mathrm{~cm}$. What is the value of $a+c$ ?

18. If n is a positive integer and D is a digit such that $\frac{n}{814}=0 . \overline{D 75}$, what is the value of n ?
19. There are 120 seats in a row. What is the fewest number of seats that must be occupied so the next person to be seated must sit next to someone?
20. If $x^{2}+\frac{1}{x^{2}}=6$ and $\mathrm{x}>0$, what is $x+\frac{1}{x}$ ? Express your answer in simplest radical form.
21. What is the probability that a randomly drawn positive factor of 60 is less than 7 ? Express your answer as a common fraction.
22. Chubby makes nonstandard checkerboards that have 15 squares on each side. The checkerboards have a black square in every corner and alternate red and black squares along every row and column. How many black squares are there on such a checkerboard?
23. Let points $\mathrm{A}=(0,0), \mathrm{B}=(1,2), \mathrm{C}=(3,3)$, and $\mathrm{D}=(4,0)$. Quadrilateral ABCD is cut into equal area pieces by a line passing through $A$. This line intersects $C D$ at point $(\mathrm{p} / \mathrm{q}, \mathrm{r} / \mathrm{s})$, where these fractions are in lowest terms. What is $\mathrm{p}+\mathrm{q}+\mathrm{r}+\mathrm{s}$ ?
24. How many rectangles are in the figure below?

25. The first four terms of an arithmetic sequence are $p, 9,3 p-q$, and $3 p+q$. What is the $2010^{\text {th }}$ term of this sequence?
26. A regular octagon $A B C D E F G H$ has sides of length two. Find the area of $\triangle A D G$.
27. Four standard, six-sided dice are to be rolled. If the product of their values turns out to be an even number, what is the probability their sum is odd? Express your answer as a common fraction.
28. Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?
29. The area of the largest equilateral triangle that can be inscribed in a square of side length 1 unit can be expressed as $a \sqrt{b}-c$, where $\mathrm{a}, \mathrm{b}$ and c are integers. What is the value of $\mathrm{a}+\mathrm{b}+\mathrm{c}$ ?
30. Fifty tickets numbered with consecutive integers are in a jar. Two are drawn at random and without replacement. What is the probability that the absolute difference between the two numbers is 10 or less? Express your answer as a common fraction.
