- 1. Compute 2024 (20*24) + (2*0*2*4).
- 2. A scout troop buys 1000 candy bars at a price of five for \$2. They sell all the candy bars at a price of two for \$1. What was their profit, in dollars?
- 3. A fair coin is tossed 3 times. What is the probability of at least two consecutive heads? Express your answer as a common fraction.
- 4. A positive number x has the property that x% of x is 4. What is x?
- 5. A point (x, y) is randomly picked from inside the rectangle with vertices (0, 0), (4, 0), (4, 1), and (0, 1). What is the probability that x < y? Express your answer as a common fraction.
- 6. Eight friends ate at a restaurant and agreed to share the bill equally. Because Judi forgot her money, each of her seven friends paid an extra \$2.50 to cover her portion of the total bill. What was the total bill, in dollars?
- 7. Let ABCD be a square. What fraction of the area of ABCD is the area of the intersection of triangles ABD and ABC?
- 8. For some positive integer k, the product $81\cdot k$ has 20 factors. Find the smallest possible value of k.
- 9. At a party, each man danced with exactly three women and each woman danced with exactly two men. Twelve men attended the party. How many women attended the party?
- 10. How many positive integer cubes divide $3! \cdot 5! \cdot 7!$?
- 11. One fair die has faces 1, 1, 2, 2, 3, 3 and another has faces 4, 4, 5, 5, 6, 6. The dice are rolled and the numbers on the top faces are added. What is the probability that the sum will be odd? Express your answer as a common fraction.
- 12. Bertha has 6 daughters and no sons. Some of her daughters have 6 daughters and the rest have none. Bertha has a total of 30 daughters and granddaughters, and no great-grand daughters. How many of Bertha's daughters and granddaughters have no daughters?
- 13. Two people wish to sit in a row of fifty chairs. How many ways can they sit in the chairs if they want to have at least two chairs between them?

- 14. Find the sum of all positive integers n does $1 + 2 + \cdots + n$ divides 6n.
- 15. Compute the number of ways to divide a 20 x 24 rectangle into 4 x 5 rectangles. (Rotations and reflections are considered distinct)
- 16. In rectangle ABCD, AD = 1, P is on \overline{AB} , and \overline{DB} and \overline{DP} trisect $\angle ADC$. What is the perimeter of $\triangle BDP$? Express your answer as a common fraction in simplest radical form.



- 17. What is the smallest positive integer that cannot be written as the sum of two nonnegative palindromic integers? (An integer is *palindromic* if the sequence of decimal digits are the same when read backwards.)
- 18. The quadratic equation $x^2 + mx + n = 0$ has roots that are twice those of $\frac{n}{2}$

 $x^2 + px + m = 0$, and none of m , n , and p is zero. What is the value of \overline{p} ?

- 19. Miki wants to distribute 75 identical candies to the students in her class such that each students gets at least 1 candy. For what number of students does Miki have the greatest number of possible ways to distribute the candies?
- 20. Let f be a function for which $f\left(\frac{x}{3}\right) = x^2 + x + 1$. Find the sum of all values of z for which f(3z) = 7. Express your answer as a common fraction.
- 21. After combining like terms, how many terms are in the expansion of $(xyz + xy + yz + xz + x + y + z)^{20}$?
- 22. An equiangular octagon has four sides of length 2 and four sides of length sqrt(2),, arranged so that no two consecutive sides have the same length. What is the area of the octagon?
- 23. When the mean, median, and mode of the list 10, 2, 5, 2, 4, 2, x are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all

possible real values of x?

- 24. Equilateral triangles ABF and BCG are constructed outside regular pentagon ABCDE. Compute angle FEG in degrees.
- 25. Compute the number of ways there are to assemble 2 red unit cubes and 25 white unit cubes into a $3 \times 3 \times 3$ cube such that red is visible on exactly 4 faces of the larger cube. (Rotations and reflections are considered distinct.)
- 26. In rectangle ABCD, points E and F lie on sides AB and CD respectively such that both AF and CE are perpendicular to diagonal BD. Given that BF and DE separate ABCD into three polygons with equal area, and that EF = 1, find the length of BD.
- 27. Mark has a cursed six-sided die that never rolls the same number twice in a row, and all other outcomes are equally likely. Compute the expected number of rolls it takes for Mark to roll every number at least once. Express your answer as a common fraction.
- 28. Inside an equilateral triangle of side length 6, three congruent equilateral triangles of side length x with sides parallel to the original equilateral triangle are arranged so that each has a vertex on a side of the larger triangle, and a vertex on another one of the three equilateral triangles, as shown below. A smaller equilateral triangle formed between the three congruent equilateral triangles has side length 1. Compute x, expressing your answer as a common fraction.



- 29. Positive integers a, b, and c have the property that a^b, b^c, and c^a end in 4, 2, and 9 respectively. Compute the minimum possible value of a+b+c.
- 30. Sally the snail sits in a 3 x 24 lattice of points (i,j) for all $1 \le i \le 3$ and $1 \le j \le 24$. She wants to visit every point in the lattice exactly once. In a move, Sally can move to a point in the lattice exactly 1 unit away. Given that Sally starts at (2,1), compute the number of possible paths Sally can take.

Answers:

- 1. 1544
 2. 100
- 3. 3/8
- 4. 20
- 5. 1/8
- 6. 140
- 7. 1/4
- 8. 8
- 9. 18
- 10. 6 11. 5/9
- 12. 26
- 12. 20
- 14. 22
- 15.6
- 15. 0 16. (6+4sqrt3)/3
- 17. 21
- 17.21
- 18. 8 19. 38
- 20. -1/9
- 21. 9261
- 22.14
- 23.20
- 24.48
- 25.114
- 26. sqrt(3)
- 27. 149/12
- 28.5/3
- 29.17
- 30. 4096