

DMC October 2019 - Hard Mock AMC 8

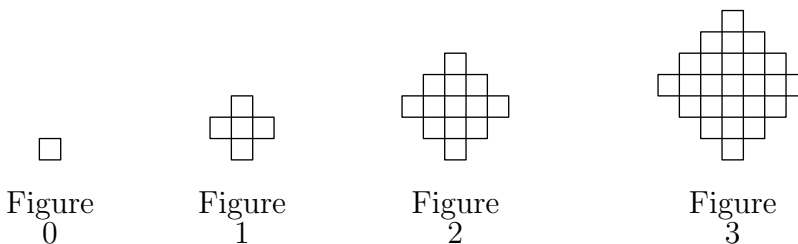
October 20, 2019

1 Problems :)

1. Bag A has three chips labeled 1, 3, and 5. Bag B has three chips labeled 2, 4, and 6. If one chip is drawn from each bag, how many different values are possible for the sum of the two numbers on the chips?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 9
2. Let $a\#b$ be defined as $ab - a - 3$. For example, $4\#5 = 20 - 4 - 3 = 13$. What is the value of $(2\#0)\#(1\#9)$?
(A) -27 (B) -23 (C) 1 (D) 5 (E) 20
3. The number of lilypads in a frogpond doubles every day. If there were 48 lilypads on Saturday, on what day of the week did the pond last have an odd number of lilypads?
(A) Monday (B) Tuesday (C) Wednesday (D) Thursday (E) Friday
4. The total in-store price for an appliance is \$99.99. A television commercial advertises the same product for three easy payments of \$29.98 and a one-time shipping and handling charge of \$9.98. How much is saved by buying the appliance from the television advertiser?
(A) 6 cents (B) 7 cents (C) 8 cents (D) 9 cents (E) 10 cents
5. Triangle ABC has altitude AH (so H lies on BC). Given that $AB = 13$, $AH = 12$, and $BC = 14$, find AC .
(A) 15 (B) $\sqrt{230}$ (C) $\sqrt{313}$ (D) 18 (E) 20
6. Alan's calculator only has the operations of multiplying by 2 and adding 1 to the number on the screen. His calculator starts with the number 2. What is the minimum number of operations that Alan has to use to get to the number 35?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8
7. Some of the problem writers and associates made the following statements:
 - Aiden: Brandon ate the cake.
 - Carla: Diva is not lying.
 - Brandon: I did not eat the cake.
 - Diva: Ellie did not eat the cake.
 - Ellie: Aiden ate the cake.If exactly one of these people is lying, who ate the cake?
(A) Aiden (B) Brandon (C) Carla (D) Diva (E) Ellie
8. The sum of two integers is 8. The sum of the squares of those two integers is 34. What is the product of the two integers?
(A) 10 (B) 12 (C) 15 (D) 18 (E) 20
9. Nick scores 91, 89, 88, 94, 87, 85 on his first 6 tests. After having his final exam, he notices that the average of all 7 of his test scores is equal to his final exam score. What was Nick's final exam score?
(A) 86 (B) 87 (C) 88 (D) 89 (E) 90

10. How many integers between 1 and 20, inclusive, that can be written as the sum of two squares of integers?
 (A) 4 (B) 5 (C) 8 (D) 12 (E) 15
11. Jake can make 12 mini cheesecakes in half an hour. If he works with his brother, Zach, they can make 84 cheesecakes in two hours. How many can Zach make in 4 hours?
 (A) 36 (B) 48 (C) 60 (D) 72 (E) 96
12. A sealed envelope contains a card with a single digit on it. Three of the following statements are true, and the other is false.
 I. The digit is 1.
 II. The digit is not 2.
 III. The digit is 3.
 IV. The digit is not 4.
 Which one of the following must necessarily be correct?
 (A) I is true. (B) I is false. (C) II is true. (D) III is true. (E) IV is false.

13. Figures 0, 1, 2, and 3 consist of 1, 5, 13, and 25 nonoverlapping squares, respectively. If the pattern were continued, how many nonoverlapping squares would there be in figure 100?



- (A) 10401 (B) 19801 (C) 20201 (D) 39801 (E) 40801
14. The Fibonacci Sequence 1, 1, 2, 3, 5, 8, 13, 21, ... starts with two 1s and each term afterwards is the sum of its predecessors. Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci Sequence?
 (A) 0 (B) 4 (C) 6 (D) 7 (E) 9
15. John makes drums that are cylinders with diameter 100cm and height 200cm. He wants to increase the volume of the drums by 44% while maintaining the same height. What length, in cm, must he make the radius of his drums in order for this to happen?
 (A) 60 (B) 72 (C) 120 (D) 144 (E) 288
16. A set with four distinct positive integer elements has smallest element 2, a range of 12, and a whole-number median. How many distinct values could be the mean of such a set?
 (A) 7 (B) 8 (C) 9 (D) 10 (E) 11
17. James is buying cookies. If he can choose chocolate chip, sugar, or peanut butter cookies, how many different ways can he buy 10 cookies?
 (A) 36 (B) 45 (C) 55 (D) 66 (E) 78
18. Nel the Frog and Ala the Kat play a game. Nel goes first, and they alternate rolling a standard 6-sided die. If they roll an even number or a number that was previously rolled, they win. What is the probability that Ala wins?
 (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{13}{36}$ (D) $\frac{5}{12}$ (E) $\frac{2}{3}$
19. Suppose $f(x)$ is a function such that $f(x) + x \cdot f(3 - x) = 2$. What is the value of $f(1)$?
 (A) -4 (B) -3 (C) -2 (D) -1 (E) 0

20. How many 6-digit numbers start with 7 and have exactly 2 identical digits?

- (A) 30240 (B) 36000 (C) 43200 (D) 45360 (E) 48000

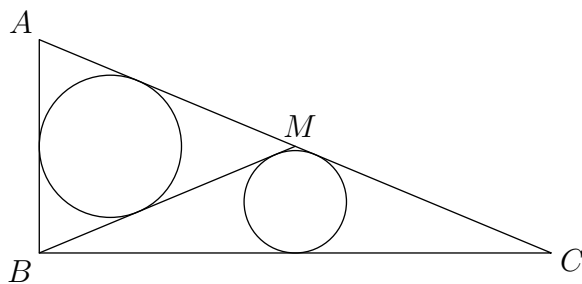
21. Jonathan purchased a bulk order of Jake's mini cheesecakes, and invites his friends Antonia, Jordan, and Amber. However, they realize that after splitting the cakes evenly, there is one cake left. To solve the problem, they invite Victor. But after splitting evenly again, there is still one cake left! Flabbergasted, they invite David and Youjung as well. But luck has it that after splitting the cakes one more time, there are now two cakes left. If Jonathan purchased more than 150 cakes, what is the minimum number of cakes he could have bought?

- (A) 161 (B) 201 (C) 221 (D) 241 (E) 261

22. Find the maximum number of elements that can be chosen from the set $\{1, 2, 3, \dots, 2013\}$ such that the sum of any two chosen elements is not divisible by 3.

- (A) 336 (B) 671 (C) 672 (D) 1342 (E) 1343.

23. ABC is a right triangle with right angle B , $AB = 10$, and $BC = 24$. Let M be the midpoint of AC . Circle O_1 is inscribed in triangle ABM , and circle O_2 is inscribed in triangle BCM . Let r_1 be the radius of O_1 , and let r_2 be the radius of O_2 . Find the value of $r_1 r_2$.



- (A) 8 (B) 10 (C) 12 (D) 14 (E) 16

24. Let x be the length of the shortest path between the points $(4, 9)$ and $(12, 4)$ that touches both the x -axis and the y -axis at least once. Find x^2 .

- (A) 89 (B) 250 (C) 425 (D) 530 (E) 614

25. Andrew the Sun Bear and Simon are 48 miles apart on an infinite flat plane. At the same time, they start running in a straight line in a random direction, with Andrew running at 20 miles per hour and Simon running at 12 miles per hour. The locus of all possible points where Andrew and Simon could collide is a circle. If Simon had instead run at x miles per hour, then the radius of this circle would be 13 miles smaller. What is the value of x ?

- (A) 6 (B) 10 (C) 16 (D) 20 (E) 24

(BONUS)

26. The probability that the triangle constructed from 3 randomly chosen points on a circle contains at least one angle less than θ , where $\theta < 45^\circ$, is $\frac{5}{9}$. What is the value of θ ?

- (A) 10° (B) 15° (C) 20° (D) 30° (E) 40°