

Team Round

Denver Math Club

January 2022

1. Roy is starting a baking company and decides that he will sell cupcakes. He sells n cupcakes for $(n+20)(n+15)$ cents. A man walks in and buys \$10.50 worth of cupcakes. Roy bakes cupcakes at a rate of 10 cupcakes an hour. How many minutes will it take Roy to complete the order?
2. A trifecta is an ordered triple of positive integers (a, b, c) with $a < b < c$ such that a divides b , b divides c , and c divides ab . What is the largest possible sum $a + b + c$ over all trifectas of three-digit integers?
3. How many six-letter words formed from the letters of AMC do not contain the substring AMC ? (For example, $AMAMMC$ has this property, but $AAMCCC$ does not.)
4. Let $ABCDEF$ be a regular hexagon. Let P be the intersection point of \overline{AC} and \overline{BD} . Suppose that the area of triangle $EF P$ is 25. What is the area of the hexagon?
5. For each integer from 1 through 2021, Tala calculated the product of its digits. Compute the sum of all 2021 of Tala's products.
6. The integers $1, 2, \dots, n$ are written in order on a long slip of paper. The slip is then cut into five pieces, so that each piece consists of some (nonempty) consecutive set of integers. The averages of the numbers on the five slips are 1234, 345, 128, 19, and 9.5 in some order. Compute n .
7. Alex is thinking of a number that is divisible by all of the positive integers 1 through 200 inclusive except for two consecutive numbers. What is the smaller of these numbers?

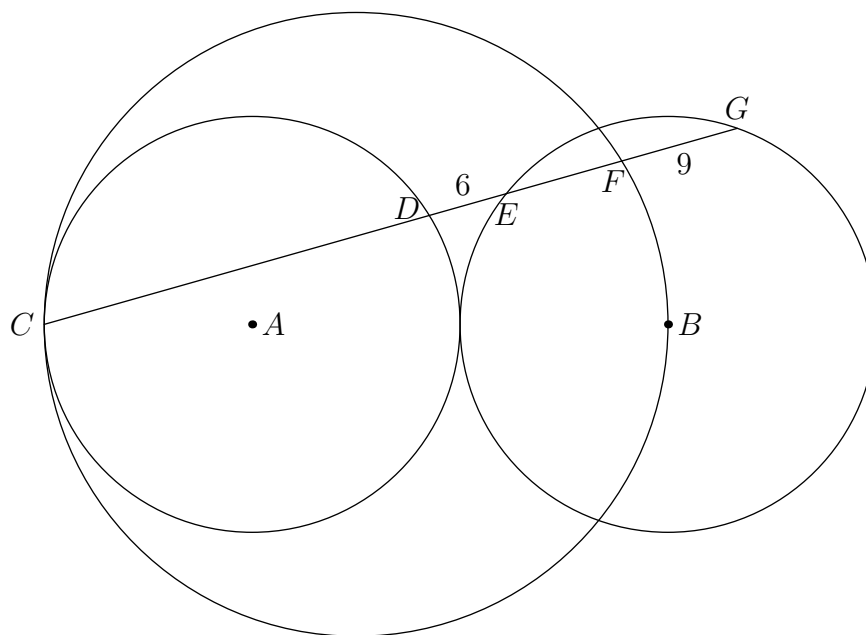
8. Two right cones each have base radius 4 and height 3, such that the apex of each cone is the center of the base of the other cone. Find the surface area of the union of the cones in terms of π .

9. Consider the base 27 number

$$n = ABCDEFGHIJKLMNOPQRSTUVWXYZ,$$

where each letter has the value of its position in the alphabet. What remainder do you get when you divide n by 100?

10. In the diagram below, the circle with center A is congruent to and tangent to the circle with center B . A third circle is tangent to the circle with center A at point C and passes through point B . Points C , A , and B are collinear. The line segment \overline{CDEFG} intersects the circles at the indicated points. Suppose that $DE = 6$ and $FG = 9$. Find AG , expressing your answer in simplest radical form.



Answers:

1.90

2.1736

3.622

4.90

5.814320

6.2014

7.127

8.62π

9.25

$10.9\sqrt{19}$