1. Suppose a and b are positive integers such that $a^b = 2^{2023}$. Compute the smallest possible value of b^a .

2. There are 800 marbles in a bag. Each marble is colored with one of 100 colors, and there are eight marbles of each color. Anna draws one marble at a time from the bag, without replacement, until she gets eight marbles of the same color, and then she immediately stops. Suppose Anna has not stopped after drawing 699 marbles. Compute the probability that she stops immediately after drawing the 700th marble.

3. The Fibonacci numbers are defined recursively by $F_0 = 0$, $F_1 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for $i \ge 2$. Given 15 wooden blocks of weights $F_2, F_3, \dots F_{16}$, compute the number of ways to paint each block either red or blue such that the total weight of the red blocks equals the total weight of the blue blocks.

4. Let ABCD be a convex quadrilateral such that $\angle ABD = \angle BCD = 90^{\circ}$, and let M be the midpoint of segment BD. Suppose that CM = 2 and AM = 3. Compute AD. Express your answer in the simplest radical form.

5. Let ABCDEF be a regular hexagon, and let P be a point inside quadrilateral ABCD. If the area of triangle PBC is 20, and the area of triangle PAD is 23, compute the area of hexagon ABCDEF.

6. Richard starts with the string DDOOOOGG. A move consists of replacing an instance of DO with OD, replacing an instance of OG with GO, or replacing an instance of GD with DG. Compute the number of possible strings he can end up with after performing zero or more moves.

7. The cells of a 5×5 grid are each colored red, white, or blue. Sam starts at the bottom-left cell of the grid and walks to the top-right cell by taking steps one cell either up or to the right. Thus, he passes through 9 cells on his path, including the start and end cells. Compute the number of colorings for which Sam is guaranteed to pass through a total of exactly 3 red cells, exactly 3 white cells, and exactly 3 blue cells no matter which route he takes.

8. Let A, X, B, and Y be points on a circle (in that order), and let line AB and line XY intersect at C. Suppose AX \cdot AY = 6, BX \cdot BY = 5, and CX \cdot CY = 4. Compute AB^2 . Express your answer as a common fraction.

Answers:

- 1. 1
- 2.99/101
- 3. 32
- 4. sqrt(21)
- 5. 189
- 6.70
- 7. 1680
- 8. 242/15