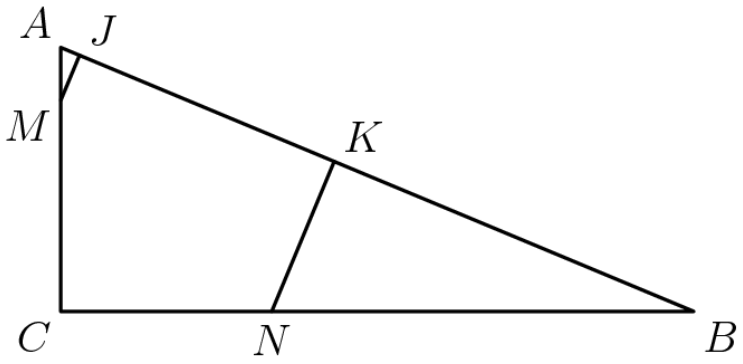
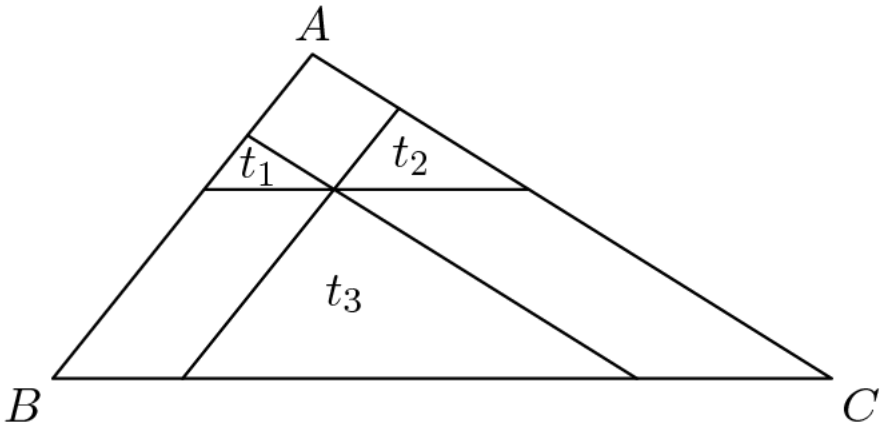
Denver Math Club
February 24, 2019
Mock State Team Round

1. The arithmetic mean of the nine numbers in the set $\{9, 99, 999, 9999, \ldots, 999999999\}$ is a 9-digit number M, all of whose digits are distinct. Which digit does the number M not contain?
2. How many ways are there to rearrange the letters in the word MISSOURI?
3. The product of three consecutive positive integers is 8 times their sum. What is the sum of their squares?
4. Ten points are marked on a circle. How many distinct convex polygons of three or more sides can be drawn using some (or all) of the ten points as vertices?
5. What is the average (mean) of all 5-digit numbers that can be formed by using each of the digits 1, 3, 5, 7, 8 and exactly once? Express your answer to the nearest tenth.
6. Two farmers agree that pigs are worth 300 dollars and that goats are worth 210 dollars. When one farmer owes the other money, he pays the debt in pigs or goats, with "change" received in the form of goats or pigs as necessary. (For example, a 390 dollar debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?
7. In $\triangle ABC$, $AB = 13$, $AC = 5$, and $BC = 12$. Points M and N lie on AC and BC, respectively, with $CM = CN = 4$. Points J and K are on AB so that MJ and NK are perpendicular to AB. What is the area of pentagon $CMJKN$? Express your answer as a common fraction.



8. Points A and B are on the parabola $y = 4x^2 + 7x - 1$, and the origin is the midpoint of AB. What is the length of AB? Express your answer in simplest radical form.
9. A point P is chosen in the interior of $\triangle ABC$ such that when lines are drawn through P parallel to the sides of $\triangle ABC$, the resulting smaller triangles t_1, t_2, and t_3 in the figure, have areas 4, 9, and 49, respectively. Find the area of $\triangle ABC$.



10. Twenty five of King Arthur's knights are seated at their customary round table. Three of them are chosen - all choices being equally likely - and are sent off to slay a troublesome dragon. What is the probability that two of the three had been sitting next to each other?