1. Given that 2 = a/20 = 60/b, what is the value of a + b?

2. Jared needs $240 to buy a new bicycle. He has saved $60 so far and saves $9 per week. In how many weeks will Jared have saved enough to buy the bicycle?

3. If a, b and c are integers such that $\frac{a+b}{2} = 3$, $\frac{b+c}{2} = 4$, and $\frac{a+c}{2} = 5$, what is the value of a + b + c?

4. If $\sqrt{x + 7} = 2 + \sqrt{x}$, what is the value of x? Express your answer as a common fraction.

5. The first two terms of a sequence are 10 and 20. If each term after the second term is the average of all of the preceding terms, what is the 2015th term?

6. What is the sum of the distinct prime factors of 2016?

7. What is the units digit of the sum of the squares of the integers from 1 to 2015, inclusive?

8. Consider an arithmetic sequence with $a_3 = 165$ and $a_{12} = 615$. For what value of n is $a_n = 2015$?

9. The length of a rectangle is three times its width. A new rectangle is created by decreasing the length of the original rectangle by 9 feet and increasing its width by 4 feet. The area of the new rectangle is the same as the area of the original rectangle. What is the perimeter of the new rectangle?

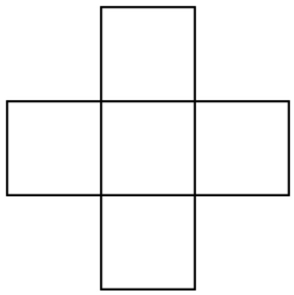
10. What is the greatest possible perimeter of an isosceles triangle with sides of length 5x + 20, 3x + 76 and x + 196?

11. At the end of 2022, Professor Lou was given a 10% pay raise from his salary at the end of 2021. However, inflation caused the worth of a dollar to decrease by 1%. If Prof. Lou's salary at the end of 2021 was worth one million dollars, how much (in dollars) was Prof. Lou's salary worth at the end of 2022? Assume that the value of the dollar has not changed.

12. A square of side length s is inscribed in circle C_1 and circumscribed about circle C_2. The area of the region in C_1 but outside C_2 is 25π. What is s?

13. Determine the largest prime which divides both $2^{24}-1$ and $2^{16}-1$.

14. Five unit squares are arranged in a plus shape as shown below:



What is the area of the smallest circle containing the interior and boundary of the plus shape, expressed as a common fraction in terms of pi?

15. We know that 201 and 9 give the same remainder when divided by 24. What is the smallest positive integer k such that $201+k$ and $9+k$ give the same remainder when divided by $24+k$?

16. Let $f(x)=x^2-kx+(k-1)^2$ for some constant k. What is the largest possible real value of k such that f has at least one real root?

17. Define $f(n)=\text{LCM}(1,2,\ldots,n)$. Determine the smallest positive integer a such that $f(a)=f(a+2)$.

18. Lthan Eou is walking around the Cartesian plane. From any point (x,y), Lthan can move to $(x+1,y)$ or $(x+1,y+3)$. How many paths can Lthan take from $(0,0)$ to $(9,9)$?

19. In a group of 2023 people, some pairs of people are friends (friendship is mutual). It is known that no two people (not necessarily friends) share a friend. What is the maximum number of unordered pairs of people who are friends?

20. Bob is flipping bottles. Each time he flips the bottle, he has a 0.25 probability of landing it. After successfully flipping a bottle, he has a 0.8 probability of landing his next flip. What is the expected value of the number of times he has to flip the bottle in order to flip it twice in a row, expressed as a common fraction?

21. Sandra the Maverick has 5 pairs of shoes in a drawer, each pair a different color. Every day for 5 days, she picks two shoes at random and puts them back in. If they are the

same color, she treats herself to a practice problem. What is the expected value (average number) of practice problems she gets to do? Express your answer as a common fraction.

22. If $ABCDE$ is a regular pentagon and X is a point in its interior such that CDX is equilateral, compute $\angle AXE$ in degrees.

23. The English alphabet, which has 26 letters, is randomly permuted. Let p1 be the probability that AB, CD, and EF all appear as contiguous substrings. Let p2 be the probability that ABC and DEF both appear as contiguous substrings. Compute p1/ p2 .

24. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

25. Square ABCD has side length 2, and X is a point outside the square such that AX = XB = $\sqrt{2}$. What is the length of the longest diagonal of pentagon AXBCD? Express your answer in the simplest radical form.

26. Emily's broken clock runs backwards at five times the speed of a regular clock. Right now, it is displaying the wrong time. How many times will it display the correct time in the next 24 hours? It is an analog clock (i.e. a clock with hands), so it only displays the numerical time, not AM or PM. Emily's clock also does not tick, but rather updates continuously.

27. Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2? Express your answer as a common fraction.

28. Let $f(x) = 1/(1-x)$. Let $f^{k+1}(x) = f(f^k(x))$, with $f^1(x) = f(x)$. What is $f^{2008}(2008)$? Express your answer as a common fraction.

29. Let ABCD be a parallelogram with AB = 480, AD = 200, and BD = 625. The angle bisector of $\angle$BAD meets side CD at point E. Find CE.

30. Let $x < 0.1$ be a positive real number. Let the foury series be $4 + 4x + 4x^2 + 4x^3 + \dots$, and let the fourier series be $4 + 44x + 444x^2 + 4444x^3 + \dots$. Suppose that the sum of the fourier series is four times the sum of the foury series. Compute x. Express your answer as a common fraction.

Answers.

1. 70
2. 20
3. 12
4. 9/16
5. 15
6. 12
7. 0
8. 40
9. 86
10. 832
11. 108900
12. 10
13. 17
14. 5pi/2
15. 8
16. 2
17. 13
18. 84
19. 1011

20. 25/4

21. 5/9

22. 84

23. 23

24. 30

25. sqrt(10)

26. 12

27. 61/81

28. -1/2007

29. 280

30. 3/40