

Hard 2020 State DMC Mock Sprint

1. In a group of 212 men and women, there were 32 more men than women. How many men were in the group?
2. The larger of two consecutive odd integers is three times the smaller. What is their sum?
3. The odds against the Patriots playing in the championship game are 4:1. The odds against the Texans playing in the championship game are 7:1. What is the probability that those two teams will play each other in the championship game? Express your answer as a common fraction.
4. The sum of three distinct primes is 40. What is their product?
5. A store normally sells windows at \$100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How many dollars will they save if they purchase the windows together rather than separately?
6. What is the smallest value of  $x^2 + 8x$  for real values of  $x$ ?
7. Points  $P$  and  $Q$  are on line segment  $AB$ , and both points are on the same side of the midpoint of  $AB$ . Point  $P$  divides  $AB$  in the ratio 2 : 3 and  $Q$  divides  $AB$  in the ratio 3 : 4. If  $PQ = 2$ , what is the length of segment  $AB$ ?
8. On a certain standardized test with 50 problems, 5 points were awarded for each correct answer, and 1 point was deducted for each incorrect answer. Alex answered all the questions on the test and scored a total of 184 points. How many questions did he answer correctly?
9. There are 292 students in a math class, of whom 90 are freshmen, 80 are math majors and 145 are neither freshmen nor math majors. How many students in the class are freshman math majors?
10. The equations  $2x + 7 = 3$  and  $bx - 10 = -2$  have the same solution for  $x$ . What is the value of  $b$ ?
11. A boat can hold three people, one of whom needs to row to cross a river that is 20 yards wide. What is the minimum distance the boat must travel to transport 9 people from the left bank of the river to the right bank?
12. Given  $(x - y)^2 = 81 - 4xy$  for positive  $x$  and  $y$ , what is the average of  $x$  and  $y$ ? Express your answer as a decimal to the nearest tenth.
13. Point  $C(4, 2)$  is on circle  $O$ , with center  $(4, -2)$ . Segment  $CD$  is a diameter of circle  $O$ . If circle  $O$  intersects the  $x$ -axis at  $A$  and  $B$ , what is the ratio of the length of  $ACB$  to the length of  $ADB$ ? Express your answer as a common fraction.
14. At the end of a professional bowling tournament, the top 5 bowlers have a playoff. First #5 bowls #4. The loser receives 5th prize and the winner bowls #3 in another game. The loser of this game receives 4th prize and the winner bowls #2. The loser of this game receives 3rd prize and the winner bowls #1. The winner of this game gets 1st prize and the loser gets 2nd prize. In how many orders can bowlers #1 through #5 receive the prizes?
15. In how many ways can the letters A, B, C and D be arranged so that no letter is adjacent to any letter that comes immediately before it or immediately after it alphabetically?
16. A wooden cube is painted on each of its faces and then cut into  $n^3$  unit cubes. If 216 of those smaller cubes are painted on exactly one face, what is the value of  $n$ ?
17. The six edges of a tetrahedron  $ABCD$  measure 7, 13, 18, 27, 36 and 41 units. If the length of edge  $AB$  is 41, what is the length of edge  $CD$ ?
18. Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?
19. Let points  $A = (0, 0)$ ,  $B = (1, 2)$ ,  $C = (3, 3)$ , and  $D = (4, 0)$ . Quadrilateral  $ABCD$  is cut into equal area pieces by a line passing through  $A$ . This line intersects  $\overline{CD}$  at point  $(x, y)$ . What is  $x + y$ ? Express your answer as a

common fraction.

20. As a used-car salesperson, Noah has a monthly sales quota, which is the minimum number of cars he must sell each month. Noah had not sold any cars in June, as of the 24th of the month. However, on June 25th, Noah sold half of the number of cars in his monthly quota, plus one more car. On June 26th, he sold half of the remaining number of cars he needed to sell, plus one more car. The same pattern continued until June 30th, when Noah sold half of the remaining cars he needed to sell, plus one more car and reached his monthly sales quota. Noah has a monthly sales quota to sell how many cars?

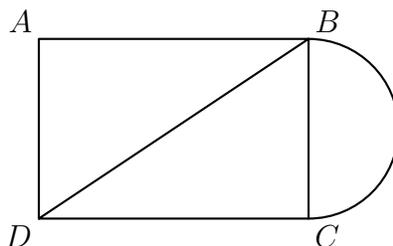
21. Twins Miley and Riley leave home at the same time, riding their bicycles in opposite directions. Miley goes northbound traveling 18 mi/h. After 2 hours, the twins are 84 miles apart. If Miley had instead been traveling eastbound, how many miles apart would the twins be after 3 hours?

22. An iterative average of the numbers 1, 2, 3, 4, and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure? Express your answer as a common fraction.

23. The point  $(5, 12)$  is rotated 90 degrees counterclockwise about the origin. What are the coordinates of its image? Express your answer as an ordered pair.

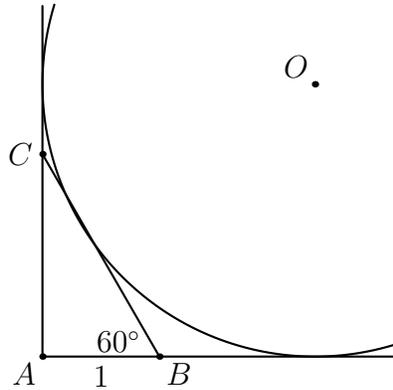
24. Rabbits Peter and Pauline have three offspring—Flopsie, Mopsie, and Cotton-tail. These five rabbits are to be distributed to four different pet stores so that no store gets both a parent and a child. It is not required that every store gets a rabbit. In how many different ways can this be done?

25. Figure ABCD can be drawn, without retracing, in one continuous pen stroke. If the stroke must begin at A, B, C or D, in how many different ways can this be done?



26. A rising number, such as 34689, is a positive integer each digit of which is larger than each of the digits to its left. There are  $\binom{9}{5} = 126$  five-digit rising numbers. When these numbers are arranged from smallest to largest, what is the sum of the digits of the 97th number in the list?

27. A circle with center  $O$  is tangent to the coordinate axes and to the hypotenuse of the  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle  $ABC$  as shown, where  $AB = 2$ . In simplest radical form, what is the radius of the circle?



28. In triangle  $ABC$ , the median from vertex  $A$  is perpendicular to the median from vertex  $B$ . If the lengths of sides  $AC$  and  $BC$  are 6 and 7 respectively, then what is the length of side  $AB$ ? Express your answer in simplest radical form.
29. What is the sum of the base-ten numbers from 5 to 50, inclusive, that are palindromes when written in base two?
30. Call a positive real number special if it has a decimal representation that consists entirely of digits 0 and 7. For example,  $\frac{700}{99} = 7.\overline{07} = 7.070707\cdots$  and  $77.007$  are special numbers. What is the smallest  $n$  such that 1 can be written as a sum of  $n$  special numbers?