1. Suppose $h \cdot a \cdot r \cdot v \cdot a \cdot r \cdot d=m \cdot i \cdot t=h \cdot m \cdot m \cdot t=100$. Find $(r \cdot a \cdot d) \cdot(t \cdot r \cdot i \cdot v \cdot i \cdot a)$.
2. Dylan the cow stands on a point $P_{0}$ inside a unit equilateral triangle $A B C$. First, he moves to $P_{1}$, the midpoint of $P_{0}$ and $A$. Then, he moves to $P_{2}$, the midpoint of $P_{1}$ and $B$. Finally, he moves to $P_{3}$, the midpoint of $P_{2}$ and $C$. Surprisingly, $P_{0}$ and $P_{3}$ are the same point! Find ${\overline{P_{1} P_{3}}}^{2}$. Express your answer as a common fraction.
3. Allen and Yang want to share the numbers $1,2,3,4,5,6,7,8,9,10$. How many ways are there to split all ten numbers among Allen and Yang so that each person gets at least one number, and either Allen's numbers or Yang's numbers sum to an even number?
4. Nine people sit down for dinner where there are three choices of meals. Three people order the beef meal, three order the chicken meal, and three order the fish meal. The waiter serves the nine meals in random order. Find the number of ways in which the waiter could serve the meal types to the nine people so that exactly one person receives the type of meal ordered by that person.
5. Let $A B C D$ be a convex quadrilateral with $A C=7$ and $B D=17$. Let $M, P, N, Q$ be the midpoints of sides $A B, B C, C D, D A$ respectively. Compute $M N^{2}+P Q^{2}$.
6. Let $a_{0}=1$ and $a_{1}=1$. For $n \geq 1$, define $a_{n+1}$ recursively to be $a_{n}+m$, where $m$ is the smallest positive integer such that $m$ is distinct from $a_{1}-a_{0}$, $a_{2}-a_{1}, a_{3}-a_{2}, \ldots a_{n}-a_{n-1}$ and that $m$ and $a_{n}$ are relatively prime. Find $a_{47}$.
7. A triple of integers $(a, b, c)$ satisfies $a+b c=2017$ and $b+c a=8$. Find the sum of all possible values of $c$.
8. Let $S_{1}=\left(\frac{1}{2}, 1\right)$. For $n \geq 2$, the set $S_{n}$ is defined as follows: if $x$ and $y$ are (possible equal) elements of $S_{n-1}$, then $\frac{2 x y}{x+y}$ is an element of $S_{n}$. Find the 2019th smallest element of $S_{12}$. Express your answer as a common fraction.
