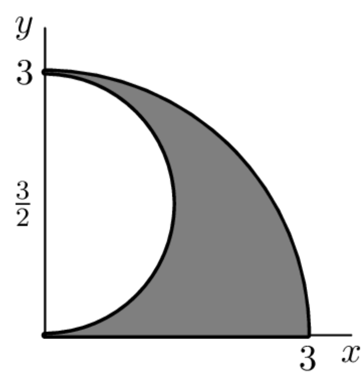
1. What is 2^(0*2*3)+0^(2*2*3)+2^(2*0*3)+3^(2*0*2)?

2. A triangle has prime number side lengths. Given that one side has length 17 and another side has length 23, how many possibilities are there for the third side length?

3. Alex places a divider in the number 20232023 so that he ends up with two new numbers. For example, he could write 2023|2023, and his new numbers would be 2023 and 2023. What is the greatest possible sum of the two new numbers?

4. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

5. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0, \frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata? Express your answer as a common fraction in terms of pi.

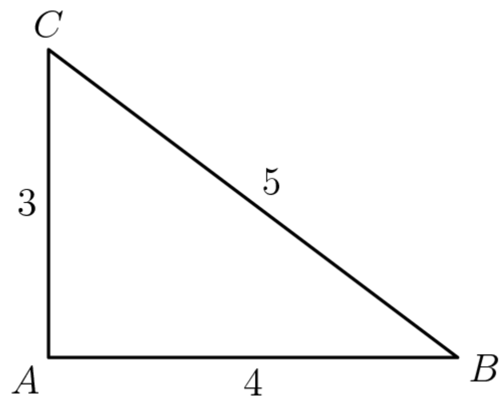


6. Let x, y, z be nonzero real numbers such that $x + y + z = xyz$. Compute $\dfrac{1+yz}{yz} + \dfrac{1+xz}{xz} + \dfrac{1+xy}{xy}$.

7. Julie runs a 2 mile route every morning. She notices that if she jogs the route 2 miles per hour faster than normal, then she will finish the route 5 minutes faster. How fast (in miles per hour) does she normally jog?

8. Kevin writes the multiples of three from 1 to 100 on the whiteboard. How many digits does he write?

9. In the figure below, choose point D on $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters. What is the area of $\triangle ABD$?

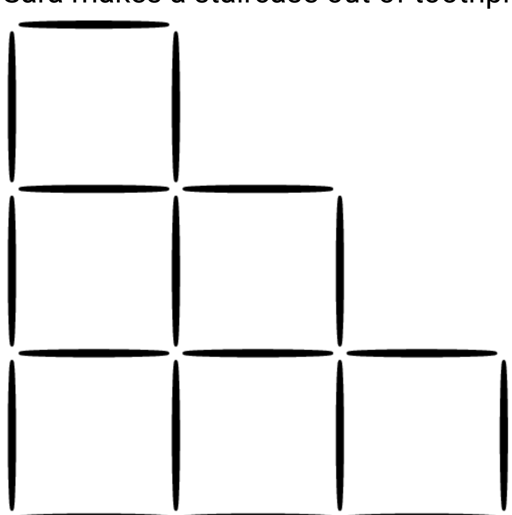


10. Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

11. For any positive integer M, the notation $M!$ denotes the product of the integers 1 through M. What is the largest integer n for which 5^n is a factor of the sum $98! + 99! + 100!$?

12. Given that x and y are positive real numbers such that $\frac{5}{x} = \frac{y}{13} = \frac{x}{y}$, find the value of $x^3 + y^3$.

13. Sara makes a staircase out of toothpicks as shown:

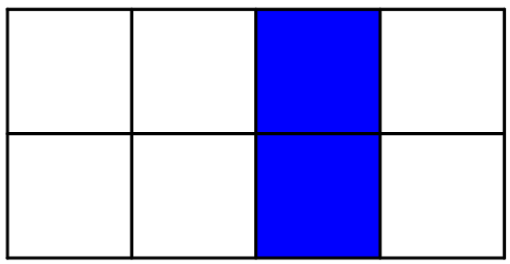


This is a 3-step staircase and uses 18 toothpicks. How many steps would be in a staircase that used 180 toothpicks?

14. In the expansion of $(2x+3y)^{20}$, find the number of coefficients divisible by 144.

15. A palindrome is a nonnegative integer that looks the same when it is written backwards. What is the smallest positive integer that cannot be expressed as the sum of two palindromes?

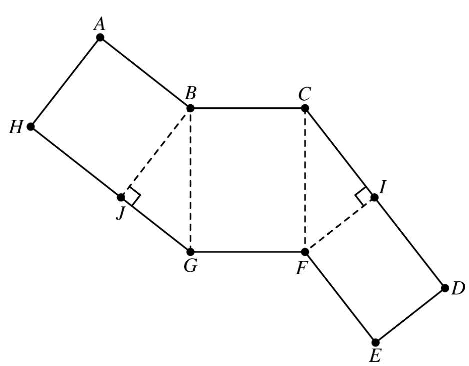
16. A random rectangle (not necessarily a square) with positive integer dimensions is selected from the 2×4 grid below. What is the probability that the selected rectangle contains a blue square, expressed as a common fraction?



17. In how many ways can the letters in $\mathbf{BEEKEEPER}$ be rearranged so that two or more $\mathbf{E}$s do not appear together?

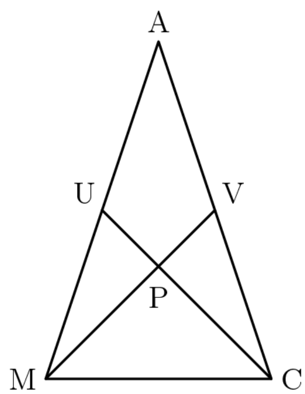
18. Seven cubes, whose volumes are 1, 8, 27, 64, 125, 216, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

19. The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that $AH = EF = 8$ and $GH = 14$. What is the volume of the prism?



20. Consider a random permutation $\{s_1, s_2, ..., s_8\}$ of $\{1, 1, 1, 1, -1, -1, -1, -1\}$. Let S be the largest of the numbers s_1, $s_1 + s_2$, $s_1 + s_2 + s_3$, ..., $s_1 + s_2 + ... + s_8$. What is the probability that S is exactly 3? Express your answer as a common fraction.

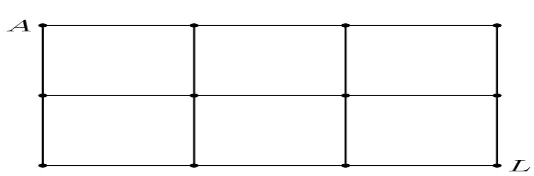
21. Triangle AMC is isosceles with $AM = AC$. Medians $\overline{MV}$ and $\overline{CU}$ are perpendicular to each other, and $MV = CU = 12$. What is the area of $\triangle AMC$?



22. A red ball and a green ball are randomly and independently tossed into bins numbered with positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball? Express as a common fraction.

23. Jared has 3 distinguishable Rolexes. Each day, he selects a subset of his Rolexes and wears them on his arm (the order he wears them does not matter). However, he does not want to wear the same Rolex 2 days in a row. How many ways can he wear his Rolexes during a 6 day period?

24. The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L, without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.)



How many different routes can Paula take?

25. Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

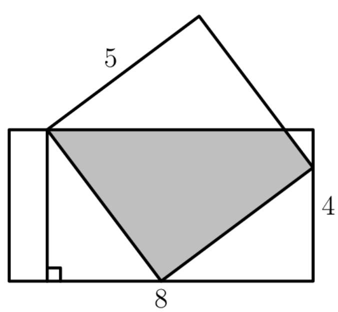
26. Let $\triangle ABC$ with $AB = AC$ and $BC = 14$ be inscribed in a circle ω. Let D be the point on ray BC such that $CD = 6$. Let the intersection of AD and ω be E. Given that $AE = 7$, find AC^2.

27. Real numbers x and y satisfy $x + y = 4$ and $x \cdot y = -2$. What is the value of

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y?$$

28. Let N be a positive multiple of 5. One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

29. The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle? Express in as mixed number.



30. Regular hexagon $ABCDEF$ has side length 2. Let G be the midpoint of $\overline{AB}$, and let H be the midpoint of $\overline{DE}$. What is the perimeter of $GCHF$? Express answer as radical in simplest form

Answers:

1. 3
2. 9
3. 20232025
4. 1/6
5. 9pi/8
6. 4
7. 6
8. 63
9. 12/5
10. 45
11. 26
12. 1170
13. 12
14. 15
15. 21
16. 3/5
17. 24
18. 658
19. 192
20. 1/10
21. 96
22. 1/3
23. 9261
24. 4

25. 11
26. 105

27. 440

28. 12
29. 15 5/8
30. 4sqrt 7