

**Denver Math Club**  
**MATHCOUNTS Set—January 2017**

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1. When Caitlin starts running towards Asha, Asha is 100 meters from Caitlin. As Caitlin runs, Asha moves directly away from Caitlin at  $\frac{1}{3}$  of Caitlin's speed. How far has Caitlin run when she first catches up to Asha? (2007 School Sprint # 24)
  
2. Point  $C$  is on the segment  $AB$  which has endpoints  $A(2, -1)$  and  $B(11, 5)$ . Point  $C$  is twice as far from point  $A$  as it is from point  $B$ . What are the coordinates of point  $C$ ? (2007 School Sprint # 21)
  
3. If  $x$  is a positive integer, what is the value of  $x$  for the equation  $\frac{x! - (x - 3)!}{23} = 1$ ? (2007 School Sprint # 16)
  
4. What is the area of the convex quadrilateral with vertices  $(-1, 0)$ ,  $(0, 1)$ ,  $(2, 0)$  and  $(0, -3)$ ? (2007 School Sprint # 20)
  
5. The four vertices of a square are  $(-5, 4)$ ,  $(1, 4)$ ,  $(1, -2)$  and  $(-5, -2)$ . What is the slope of the line that passes through the origin and divides the square into two congruent trapezoids? Express your answer as a common fraction. (2014 School Sprint #29)
  
6. An arithmetic progression is formed by five distinct prime numbers. What is the least possible sum of those five numbers? (2014 School Sprint #24)
  
7. Amy has \$5 in quarters, dimes and nickels, where the total value in quarters is more than the total value of the dimes, and the total value of the dimes is more than the total value of the nickels. What is the maximum number of coins Amy could have? (2014 School Sprint #28)
  
8. There are six points in a plane, and no three of these points are collinear. How many distinct lines are determined by all possible pairs of these six points? (2014 Chapter Sprint #11)
  
9. On the ceiling of a cavern, a stalactite grows downward 0.004 *inches* per year. Ten *feet* directly below on the cavern's floor, a stalagmite grows upward 0.006 *inches* per year. How many years will it take them to meet? (2014 Chapter Sprint # 14)
  
10. What is the smallest prime number that divides some number of the form  $424242 \cdots 42 \pm 1$ ? (2014 Chapter Sprint #28)

11. If  $\frac{1}{x} + \frac{3}{y} = \frac{3}{4}$  and  $\frac{3}{x} - \frac{2}{y} = \frac{5}{12}$ , what is  $x + y$ ? (2014 Chapter Sprint #27)
12. What is the correct ordering of the three numbers  $\frac{5}{19}$ ,  $\frac{7}{21}$ , and  $\frac{9}{23}$ , in increasing order? (2012 AMC 8 #20)
13. Marla has a large white cube that has an edge of 10 feet. She also has enough green paint to cover 300 square feet. Marla uses all the paint to create a white square centered on each face, surrounded by a green border. What is the area of one of the white squares, in square feet? (2012 AMC 8 #21)
14. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50? (2012 AMC 8 #18)
15. In how many ways can 10001 be written as the sum of two primes? (2011 AMC 8 #24)
16. What is the tens digit of  $7^{2011}$ ? (2011 AMC 8 #22)
17. What is the correct ordering of the three numbers,  $10^8$ ,  $5^{12}$ , and  $2^{24}$ ? (2010 AMC 8 #24)
18. Andy and Bethany have a rectangular array of numbers with 40 rows and 75 columns. Andy adds the numbers in each row. The average of his 40 sums is  $A$ . Bethany adds the numbers in each column. The average of her 75 sums is  $B$ . What is the value of  $\frac{A}{B}$ ? (2009 AMC 8 #21)
19. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3 or 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3? (2007 AMC 8 #24)
20. What is the value of  $\sqrt{1 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}}$ , where all subsequent numbers are 2? (2014 Chapter Sprint #30)