

Denver Math Club

July 2017

1. If 7 clips equal 8 claps and 12 claps equal 5 clups, how many clups are equal to 168 clips?
2. A bag contains 94 red marbles, 9 green marbles, 16 blue marbles and 71 black marbles. What is the minimum amount of marbles that can be chosen to guarantee getting at least two marbles with different colors?
3. The length of a rectangle is 44 less than thrice its width. If the area of the rectangle is 119, what is the width of the rectangle?
4. An operation between two nonzero real numbers  $a$  and  $b$  is defined as  $a \diamond b = ab - b$ . If  $a \diamond (b \diamond a) = (b \diamond a) \diamond a$ , what is the minimum value of  $b^2 + a^2$ ?
5. What is the units digit of  $3^{3^{3^3}}$ ?
6. 5 distinct points are placed on each side of a square. Each point is connected exactly once to one other point with a line segment. If  $m$  is the minimal number of intersections between the segments in the square and  $M$  is the maximal number, what is  $M - m$ ?
7. If  $p^4 + 1$  is a prime number for a prime number  $p$ , what is the value of  $p^4 + 1$ ?
8. A caterpillar crawls on the coordinate plane starting from the origin. If the caterpillar crawls one step upward with probability  $\frac{1}{4}$  and one step to the right with probability  $\frac{3}{4}$ , what is the probability that the caterpillar never reaches the point  $(2, 2)$ ?
9. A die with 9 faces consists of a square pyramid glued to a cube. All the edges of the die have length one, and the pyramid and the cube are glued together along the shared square face. When this die is rolled and lands on one of its triangular faces, how high off the ground is the highest corner on the die?
10. In how many ways can I receive \$20.18 in change if I can only receive pennies, dimes and nickels such that the number of pennies I receive can be equally distributed among seven people?