1. Convert 1827462_9 to a base 3 numeral.

2. By a proper divisor of a natural number we mean a positive integral divisor other than 1 and the number itself. A natural number greater than 1 will be called "nice" if it is equal to the product of its distinct proper divisors. What is the sum of the first ten nice numbers?

3. Let a binary operation $*$ on ordered pairs of integers be defined by $(a, b) * (c, d) = (a - c, b + d)$. Then, if $(3, 2) * (0, 0)$ and $(x, y) * (3, 2)$ represent identical pairs, x equals...?

4. If $F_1 = 1$ and $F_n = 2F_{n-1} + 1$ then for how many values of n such that $1 \le n \le 100$ is F_n divisible by 3?

5. What is the greatest prime factor of $12! + 14!$?

6. What is the measure of the angle formed by the hands of a clock at $4 : 14$?

7. In a pickle ball series in which the Mathcounters are playing the AIMErs, the winning team is the first team to win 5 matches. Each team has a $\frac{1}{2}$ probability of winning any game. If the series lasts 7 games, what is the probability that the Mathcounters win?

8. For any integer n greater than 1 and less than 1000, what is the maximum number of prime numbers greater than $n! + 1$ and less than $n! + n$?

9. In triangle ABC, $\measuredangle CBA = 72°$, E is the midpoint of side AC, and D is a point on side BC such that $2BD = DC$; AD and BE intersect at F. The ratio of the area of triangle BDF to the area of quadrilateral $FDCE$ is

10. Triangle PAB and square $ABCD$ are in perpendicular planes. Given that $PA = 3, PB = 4$, and $AB = 5$, what is PD?

11. A function f from the integers to the integers is defined as follows:

$$f(n) = \begin{cases} n + 3 & \text{if n is odd} \\ n/2 & \text{if n is even} \end{cases}$$

Suppose k is odd and $f(f(f(k))) = 27$. What is the sum of the digits of k?

12. If n is a positive integer such that $2n$ has 28 positive divisors and $3n$ has 30 positive divisors, then how many positive divisors does $6n$ have?

13. What is the area of the polygon whose vertices are the points of intersection of the curves $x^2 + y^2 = 25$ and $(x - 4)^2 + 9y^2 = 81$?

14. Two parabolas have equations $y = x^2 + ax + b$ and $y = x^2 + cx + d$, where a, b, c, and d are integers (not necessarily different), each chosen independently by rolling a fair six-sided die. What is the probability that the parabolas have at least one point in common?

15. Let ABC be a triangle with $\angle BAC = 117°$. The angle bisector of $\angle ABC$ intersects side AC at D. Suppose $\triangle ABD \sim \triangle ACB$. Compute the measure of $\angle ABC$, in degrees.

16. How many 4-digit positive integers are there in which each digit after the leftmost digit is smaller than the digit to its left?

17. Kristin and Ben own a cycle shop. They sell bicycles and tricycles. On Tuesday, they got a new shipment of cycles that needed to be assembled. In the shipment there were 28 seats and 67 wheels. How many tricycles needing assembly arrived in Tuesday's shipment?

18. A store sells pencils in groups of 3 or 7. How many positive integer number of pencils cannot be bought at this store? (For example, 1 pencil cannot, but 10 = 3+7 can).

19. What is the largest integral value of n such that 213! is divisible by $(8!)^n$?

20. What is the smallest positive integer than 10,920 can be multiplied by to produce a perfect square?

21. A triangle has two sides of length 14, 17. What is the minimum possible integral value for the length of the third side such that the triangle is obtuse?

22. A rectangular prism has side lengths x, y, z with $x^2 + y^2 + z^2 = 446$, volume 1638, and surface area 850. What is the volume of a new rectangular prism with side lengths $x + 2, y + 2, z + 2$?

23. For a positive integer $n > 2$, consider the $n - 1$ fractions

$$\frac{2}{1}, \frac{3}{2}, \cdots, \frac{n}{n-1}$$

The product of these fractions equals n, but if you reciprocate (i.e. turn upside down) some of the fractions, the product will change. What is the sum of the 4 smallest such values of n such that you can make the product equal 1?

24. In square $EFGH$, two semicircles are inscribed with $\overline{FG}$ and $\overline{EH}$ as diameters and centered at the midpoints of the sides. Points C and D lie on sides EH and FG, respectively, such that $3EC = HC$ and $3FD = DG$. The segment CD intersects the two semicircles at A and B. What is the ratio of the length of AB to the length of CD? Express your answer as a common fraction in simplest radical form. 25. What is the greatest possible product of three distinct prime numbers whose sum is 100? 26. If a coin is flipped five times, what is the probability that heads does not come up two flips in a row? Express your answer as a common fraction. 27. Let $n = 2^{31}3^{19}$. How many positive integer divisors of n^2 are less than n but do not divide n?

28. Two distinct circles and a square lie in the plane. What is the maximum number of points of intersection of two or more of these three figures? 29. A man is running through a train tunnel. When he is $\frac{2}{5}$ of the way through, he hears a train that is approaching the tunnel from behind him at a speed of 60 mph. Whether he runs ahead or runs back, he will reach an end of the tunnel at the same time the train reaches that end. At what rate, in mph, is he running? (Assume that he runs at a constant rate).

30. What is the value of $2\sqrt{1.75 + \sqrt{3}} - (1.75 + \sqrt{3})$? Express your answer as a decimal to the nearest hundredth.