1. How many positive integers have distinct prime digits?

2. 1 red marble and some green marbles are placed in a jar. 7 of them are picked at random. The probability that the red marble is picked is 1/7. How many green marbles were in the jar?

3. Alice rolls a fair six-sided die with the numbers 1 through 6, and Bob rolls a fair eight-sided die with the numbers 1 through 8. Alice wins if her number divides Bob's number, and Bob wins otherwise. What is the probability that Alice wins?

4. Kurtis' school schedule is made up of four classes, followed by lunch, followed by three more classes. In how many ways can Kurtis arrange his schedule if two of his classes (Reading and Writing) must occur one immediately after the other?

5. Five people, named $A, B, C, D,$ and E, are standing in line. If they randomly rearrange themselves, what's the probability that nobody is more than one spot away from where they started?

6. On a math test, Alice, Bob, and Carol are each equally likely to receive any integer score between 1 and 10 (inclusive). What is the probability that the average of their three scores is an integer?

7. Ben flips a coin 10 times and then records the absolute difference between the total number of heads and tails he's flipped. He then flips the coin one more time and records the absolute difference between the total number of heads and tails he's flipped. If the probability that the second number he records is greater than the first can be expressed as $\frac{a}{b}$ for positive integers a, b with $\gcd(a, b) = 1$, then find $a + b$.

8. There are 10 balls in a bucket, and there are 5 colors. Each color has exactly 2 balls of that color. Every time a ball is selected uniformly, randomly, and independently from the bucket, its color is noted and the ball is replaced. What is the expected number of selections from the bucket until one ball of every color has been seen? Express your answer as a common fraction.

9. Six people are playing poker. At the beginning of the game, they have 1, 2, 3, 4, 5, and 6 dollars, respectively. At the end of the game, nobody has lost more than a dollar, and each player has a distinct nonnegative integer dollar amount. (The total amount of money in the game remains constant.) How many distinct finishing rankings (i.e. lists of first place through sixth place) are possible

10. In any finite grid of squares, some shaded and some not, for each unshaded square, record the number of shaded squares horizontally or vertically adjacent to it; this grid's *score* is the sum of all numbers recorded this way. Deyuan shades each square in a blank $n \times n$ grid with probability k; he notices that the expected value of the score of the resulting grid is equal to k, too! Given that $k > 0.9999$, find the minimum possible value of n.

Answers:

1. 24
2. 48
3. 3/8
4. 1200
5. 1/15
6. 167/500
7. 831
8. 137/12
9. 34
10. 51