1. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$\underline{1}\ \underline{A}\ \underline{2}$. What is the missing digit A of this 3-digit number?

2. Each of two boxes contains three chips numbered 1, 2, 3. A chip is drawn randomly from each box and the numbers on the two chips are multiplied. What is the probability that their product is even?

3. What is the units digit of the product of all the odd positive integers strictly less than 2023?

4. How many integers between 1000 and 9999 have four distinct digits?

5. An integer N is selected at random in the range $1 \leq N \leq 2020.$ What is the probability that the remainder when N^{16} is divided by 5 is 1?

6. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

7. A single bench section at a school event can hold either 7 adults or 11 children. When N bench sections are connected end to end, an equal number of adults and children seated together will occupy all the bench space. What is the least possible positive integer value of N?

8. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white. If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

9. Three members of the Euclid Middle School girls' softball team had the following conversation.
   Ashley: I just realized that our uniform numbers are all 2-digit primes.
   Bethany: And the sum of your two uniform numbers is the date of my birthday earlier this month.
   Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.
   Ashley: And the sum of you two uniform numbers is today's date.

   What number does Caitlin wear?

10. Ms. Carr asks her students to read any 5 of the 10 books on a reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?

Answers:

1. 3
2. 5/9
3. 5
4. 4536
5. 4/5
6. 5
7. 18
8. 5/54
9. 11
10. 25/63

1. The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Eight" teams?

2. The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k?

3. Alice refuses to sit next to either Bob or Carla. Derek refuses to sit next to Eric. How many ways are there for the five of them to sit in a row of 5 chairs under these conditions?

4. The number $21! = 51,090,942,171,709,440,000$ has over $60,000$ positive integer divisors. One of them is chosen at random. What is the probability that it is odd?

5. Let $N = 123456789101112\ldots4344$ be the 79-digit number obtained that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

6. The decimal representation of $\dfrac{1}{20^{20}}$ consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?

7. Nine delegates, three each from three different countries, randomly select chairs at a round table that seats nine people. Find the probability that each delegate sits next to at least one delegate from another country.

8. Let R be the set of all possible remainders when a number of the form 2^n, where n is a nonnegative integer, is divided by 1000. Find the sum of all elements in R.

9. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

10. Six men and some number of women stand in a line in random order. Let p be the probability that a group of at least four men stand together in the line, given that every man stands next to at least one other man. Find the least number of women in the line such that p does not exceed 1 percent.

Answers:

1. 88
2. 991
3. 28
4. 1/19
5. 9
6. 26
7. 41/56
8. 6207
9. 226
10. 594