1. Define the *digital reduction* of a two-digit positive integer $\underline{AB}$ to be the quantity $\underline{AB} - A - B$. Find the greatest common divisor of the digital reductions of all the two-digit positive integers. (For example, the digital reduction of $62$ is $62 - 6 - 2 = 54.$)

2. For each positive integer n between $1$ and $1000$ (inclusive), Ben writes down a list of $n$'s factors, and then computes the median of that list. He notices that for some $n$, that median is actually a factor of $n$. Find the largest $n$ for which this is true.

3. The six-digit number $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{A}$ is prime for only one digit $A$. What is $A$?

4. Let k = 20^20. Suppose that 20^k / k^20 = 20^n . Find the largest power of 20 that divides n.

5. How many of the first ten numbers of the sequence $121$, $11211$, $1112111$, ... are prime numbers?

6. Find the largest integer $n$ for which $\dfrac{101^n + 103^n}{101^{n-1} + 103^{n-1}}$ is an integer.

7. What is the largest prime $p$ for which the numbers $p^2 - 8$, $p^2 - 2$, and $p^2 + 10$ are all prime as well?

8. Let $D$ be the set of positive divisors of $700$. The nonempty subsets of $D$ with an even sum can be expressed as a^b-c, where a,b,c are positive integers with a as small as possible. Find a+b+c.

9. For any prime number $p$, let $S_p$ be the sum of all the positive divisors of $37^p p^{37}$ (including $1$ and $37^p p^{37}$). Find the sum of all primes $p$ such that $S_p$ is divisible by $p$.

10. When $15!$ is converted to base $8$, it is expressed as $\overline{230167356abc00}$ for some digits $a$, $b$, and $c$. Find the missing string $\overline{abc}$.

Answers:

1. 9
2. 961
3. 9
4. 400
5. 0
6. 1
7. 7
8. 20
9. 19
10. 540