1. In rectangle $A B C D, A B=6$ and $A D=8$. Point $M$ is the midpoint of $\overline{A D}$. What is the area of $\triangle A M C$ ?
2. Rectangle $D E F A$ below is a $3 \times 4$ rectangle with $D C=C B=B A$. What is the area of the "bat wings"?

3. The $y$-intercepts, $P$ and $Q$, of two perpendicular lines intersecting at the point $A(6,8)$ have a sum of zero. What is the area of $\triangle A P Q$ ?
4. A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle? Express your answer as a common fraction.

5. Two equilateral triangles are contained in a square whose side length is $2 \sqrt{3}$. The bases of these triangles are the opposite sides of the square, and their intersection is a rhombus.

What is the area of the rhombus, expressed in simplest radical form?
6. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger? Express your answer as a common fraction in simplest radical form.
7. A circle is circumscribed around an isosceles triangle whose two congruent angles have degree measure $x$. Two points are chosen independently and uniformly at random on the circle, and a chord is drawn between them. The probability that the chord intersects the 14
triangle is $\overline{25}$. Find the difference between the largest and smallest possible values of $x$.
8. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object? Express your answer as a common fraction in simplest radical form.
9. Circle $C$ with radius 2 has diameter $\overline{A B}$. Circle $D$ is internally tangent to circle $C$ at $A$. Circle $E$ is internally tangent to circle $C$, externally tangent to circle $D$, and tangent to $\overline{A B}$. The radius of circle $D$ is three times the radius of circle $E$. What is the radius of circle D , expressed in simplest radical form?
10. A block of wood has the shape of a right circular cylinder with radius 6 and height 8 , and its entire surface has been painted blue. Points $A$ and $B$ are chosen on the edge on one of the circular faces of the cylinder so that $\overparen{A B}$ on that face measures $120^{\circ}$. The block is then sliced in half along the plane that passes through point $A$, point $B$, and the center of the cylinder, revealing a flat, unpainted face on each half. What is the area of one of those unpainted faces? Express your answer in simplest radical form in terms of pi.


Answers:

1. 12
2. 3
3. 60
4. $120 / 17$
5. 8sqrt3-12
6. sqrt10/10
7. 48
8. 2 sqrt3/3
9. 4sqrt15-14
10. $20 \mathrm{pi}+30$ sqrt3
