DMC Mock AMC 8

October 2017

1. Lucy takes 3 hours and 20 minutes to finish a horrible history project. How long in seconds did the project take her?

(A) 200 **(B)** 320 **(C)** 12000 **(D)** 19200 **(E)** 720000

2. If $a@b = \frac{a \times b}{a+b}$, for a, b positive integers, then what is 5@10?

(A) $\frac{3}{10}$ **(B)** 1 **(C)** 2 **(D)** $\frac{10}{3}$ **(E)** 50

3. The three-digit number $2a3$ is added to the number 326 to give the three-digit number $5b9$. If $5b9$ is divisible by 9, then $a + b$ equals:

(A) 2 **(B)** 4 **(C)** 6 (D) 8 (E) 9

4. Joe has a certain number of blocks which he arranges in a rectangular array with no leftover blocks. He then is able to arrange the blocks in another rectangular array, this one with 1 fewer rows and 2 extra columns. Which of the following is a possible number of blocks Joe could have?

(A) 20 **(B)** 42 **(C)** 64 **(D)** 112 **(E)** 162

5. The perimeter of a square and of a circle are equal. If the square has area 64, what is the diameter of the circle?

(A) 8π **(B)** 16 **(C)** $\frac{16}{\pi}$ **(D)** $\frac{32}{\pi}$ **(E)** 64π

6. Mo squares a number x, adds the original number x to his result, and gets a number equal to the square of the number that is one more than his original number. What is x?

(A) -1 **(B)** $-1/2$ **(C)** 0 **(D)** 1 **(E)** $\frac{1+\sqrt{5}}{2}$

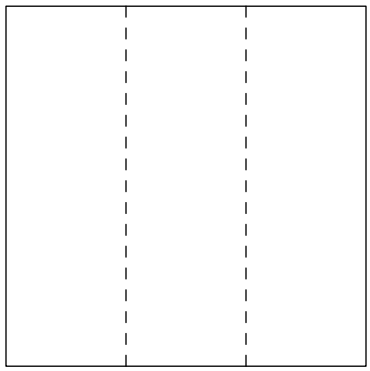
7. A father takes his twins and a younger child out to dinner on the twins' birthday. The restaurant charges \$4.95 for the father and \$0.45 for each year of a child's age, where age is defined as the age at the most recent birthday. If the bill is \$9.45, which of the following could be the age of the youngest child?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?
(A) 3 **(B)** 3.125 **(C)** 3.5 **(D)** 4 **(E)** 4.5

9. If a, b and c are positive integers and a and b are odd, then $3^a + (b-1)^2 c$ is
(A) *odd for all choices of c*
(B) *even for all choices of c*
(C) *odd, if c is even; even, if c is odd*
(D) *odd, if c is odd; even, if c is even*
(E) *odd, if c is not a multiple of 3 ; even if c is a multiple of 3*

10. A square is cut into three rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three rectangles is 24, then the area of the original square is



(A) 24 **(B)** 36 **(C)** 64 **(D)** 81 **(E)** 96

11. Marsha adds all but one of the first ten positive integers. Her sum is a square number. Which one of the first ten positive integers did Marsha not include? **(A)** 1 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

12. What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?
(A) 110 **(B)** 165 **(C)** 330 **(D)** 625 **(E)** 660

13. If x, y, and z are positive with $xy = 24$, $xz = 48$, and $yz = 72$, then $x + y + z$ is
(A) 18 **(B)** 19 **(C)** 20 **(D)** 22 **(E)** 24

14. In a meeting of philosophers and mathematicians, one out of every seven mathematicians is a philosopher, and one out of every nine philosophers is a

mathematician. If there are 24 people at the meeting who are mathematicians but not philosophers, how many more philosophers than mathematicians are at the meeting?

(A) 3 **(B)** 8 **(C)** 10 **(D)** 48 **(E)** 64

15. How many zeroes are there at the end the number $9^{999} + 1$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 998 **(E)** 999

16. Joe picks three distinct integers from $\{1, 2, 3, 4, 5, 6\}$, and Jack takes the remaining three. What is the probability that the product of Joe's integers is greater than the product of Jack's?

(A) 0 **(B)** $\frac{1}{3}$ **(C)** $\frac{7}{20}$ **(D)** $\frac{1}{2}$ **(E)** 1

17. Two eight-sided dice each have faces numbered 1 through 8. When the dice are rolled, each face has an equal probability of appearing on the top. What is the probability that the product of the two top numbers is greater than their sum?

(A) $\frac{1}{2}$ **(B)** $\frac{47}{64}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{55}{64}$ **(E)** $\frac{7}{8}$

18. A wooden cube n units on a side is painted red on all six faces and then cut into n^3 unit cubes. Exactly one-fourth of the total number of faces of the unit cubes are red. What is n?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

19. In $\triangle ABC$, $AB = 13$, $BC = 14$ and $CA = 15$. Define M as the midpoint of side AB and H as the foot of the altitude from A to BC. The length of HM is

(A) 6 **(B)** 6.5 **(C)** 7 **(D)** 7.5 **(E)** 8

20. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. The aquarium is tilled with water to a depth of 37 cm. A rock with volume $1000cm^3$ is then placed in the aquarium and completely submerged. By how many centimeters does the water level rise?

(A) 0.25 **(B)** 0.5 **(C)** 1 **(D)** 1.25 **(E)** 2.5

21. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

(A) 1 **(B)** 4 **(C)** 36 **(D)** 49 **(E)** 81

22. In the multiplication problem below, A, B, C and D are different digits. What is $A + B$?

$$\begin{array}{cccc} & A & B & A \\ \times & & C & D \\ \hline C & D & C & D \end{array}$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 9

23. Six lilypads are arranged in a circle with a frog on each. Every minute, exactly one frog jumps from the lilypad it is on to one of the two adjacent lilypads. For many numbers n between 1 and 100, inclusive, is it possible that after n minutes, all the frogs are on the same lilypad?

(A) 0 **(B)** 25 **(C)** 46 **(D)** 50 **(E)** 91

24. How many ordered pairs of integers (x, y) are there such that

$$0 < |xy| < 36?$$

(A) 100 **(B)** 144 **(C)** 216 **(D)** 444 **(E)** 524

25. The sequence $1, 2, 1, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, \ldots$ consists of 1's separated by blocks of 2's with n 2's in the nth block. The sum of the first 1234 terms of this sequence is

(A) 1996 **(B)** 2419 **(C)** 2429 **(D)** 2439 **(E)** 2449