

1. What is $(202)(3) + (20)(23) + (20)(23) + (202)(3) - 2023$?
A) 2023 B) 218 C) 109 D) 202 E) 101
2. Which of the following numbers is not a perfect square?
A) 1^{2016} B) 2^{2017} C) 3^{2018} D) 4^{2019} E) 5^{2020}
3. Alex recites the multiples of 5 starting from -15 and going up. Bob recites the multiples of 3 starting at 111 and going down. If Alex and Bob say numbers at the same rate, which number (if any) do they say simultaneously?
A) 18 B) 30 C) 15 D) 25 E) None
4. Jefferson Middle School has the same number of boys and girls. $\frac{3}{4}$ of the girls and $\frac{2}{3}$ of the boys went on a field trip. What fraction of the students on the field trip were girls?
A) $\frac{1}{2}$ B) $\frac{9}{17}$ C) $\frac{7}{13}$ D) $\frac{2}{3}$ E) $\frac{14}{15}$
5. A regular polygon has 54 diagonals. How many sides does it have?
A) 6 B) 9 C) 10 D) 12 E) 15
6. How many positive factors does 23232 have?
A) 9 B) 12 C) 28 D) 36 E) 42
7. Katie normally takes 20 minutes per homework assignment. However, if her friends are distracting her, it takes her 30 minutes per homework assignment. Given that Katie spent two hours and completed 5 homework assignments, for how many minutes were her friends distracting her?
A) 30 B) 48 C) 60 D) 72 E) 90
8. How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?
A) 1 B) 2 C) 3 D) 4 E) 5
9. How many ways are there to arrange the letters of BANANA such that at least five of the letters are in alphabetical order?
A) 9 B) 11 C) 13 D) 15 E) 17
10. The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number

QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P ?

A) 1 B) 2 C) 3 D) 4 E) 5

11. How many ordered pairs (x,y) satisfy $x^2/16+y^2/9 = |x|/10+|y|/3=1$?

A) 0 B) 2 C) 4 D) 6 E) 8

12. Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D . The circles intersect at two points, one of which is E . What is the degree measure of $\angle CED$?

A) 90 B) 105 C) 120 D) 135 E) 150

13. An equilateral triangle of side length 10 is divided into 100 equilateral triangles of side length 1. The perimeter of the large equilateral triangle is colored red. A random equilateral triangle of side length 1 is selected, and a random side from this equilateral triangle is chosen. What is the probability that the side is painted red?

A) 1/10 B) 1/8 C) 1/6 D) 1/4 E) 1/2

14. Evaluate the sum $11^2 - 1^1 + 12^2 - 2^2 + 13^2 - 3^2 + \cdots + 20^2 - 10^2$.

A) 2000 B) 2100 C) 2200 D) 2300 E) 2400 F) 2500

15. Find the minimum value of $(x+24/y)(y+6/x)$ over all positive reals x and y .

A) 52 B) 54 C) 56 D) 58 E) 60

16. The side lengths of a triangle are distinct positive integers. One of the side lengths is a multiple of 42, and another is a multiple of 72. What is the minimum possible length of the third side?

A) 2 B) 5 C) 7 D) 10 E) 12

17. Nick chooses a random real number x between 0 and 360. Then, Nick makes a sector of radius 100 and central angle x degrees. Nick folds this sector into a cone. Which of the following is closest to the probability that the height of the cone is greater than 80?

A) 40% B) 47% C) 50% D) 53% E) 60%

18. Find the number of triples (a, b, c) of positive integers such that $a + ab + abc = 11$.

A) 1 B) 2 C) 3 D) 4 E) 5

19. Abby wrote the numbers from 1 to 20 on a blackboard, except for one. Carl then wrote down all divisors of the product of the numbers on the blackboard. Given that the number of numbers Carl wrote down is divisible by 133, what is the sum of the numbers Abby wrote down?
A) 206 B) 203 C) 201 D) 194 E) 191
20. There are 15 stones placed in a line. In how many ways can you mark 5 of these stones so that there are an odd number of stones between any two of the stones you marked?
A) 21 B) 35 C) 56 D) 77 E) 98
21. Ethan starts with the string "01." In a move, Ethan can add the string "01" into any position of the current string. For example, after one move, Ethan could end up with either 0101 or 0011. Which of the following strings could Ethan create using some sequence of moves?
A) 0110100101 B) 0101101001 C) 0100111001 D) 0100101101 E) 0100011110
22. Marcus and four of his relatives are at a party. Each pair of the five people are either *friends* or *enemies*. For any two enemies, there is no person that they are both friends with. In how many ways is this possible?
A) 52 B) 72 C) 95 D) 108 E) 120
23. Triangle ABC satisfies $AB = 8$, $BC = 7$, $CA = 9$. Point E lies on segment AC such that BE bisects angle ABC, and point F lies on segment AB such that CF bisects angle ACB. What is the area of triangle AEF?
A) $(10\sqrt{3})/3$ B) $(16\sqrt{3})/3$ C) $(12\sqrt{5})/5$ D) $(18\sqrt{5})/5$ E) $(20\sqrt{7})/7$
24. Let $\triangle ABC$ be an equilateral triangle with height 13 , and let O be its center. Point X is chosen at random from all points inside $\triangle ABC$. Given that the circle of radius 1 centered at X lies entirely inside $\triangle ABC$, what is the probability that this circle contains O ?
A) $(\pi\sqrt{3})/100$ B) $(\pi\sqrt{3})/50$ C) $(3\pi\sqrt{3})/100$ D) $(\pi\sqrt{3})/25$ E) $(\pi\sqrt{3})/20$
25. A unitary divisor of n is a divisor d such that $\gcd(d, n/d) = 1$. A bi-unitary divisor of n is a divisor d such that 1 is the only unitary divisor shared by d and n/d . How many bi-unitary divisors does $10!$ have?
A) 128 B) 144 C) 168 D) 192 E) 270

Answers:

1. C

2. B

3. E

4. B

5. D

6. E

7. C

8. E

9. C

10. A

11. D

12. C

13. A

14. B

15. B

16. C

17. E

18. C

19. C

20. D

21. D

22. A

23. D

24. A

25. A