1. Jamar bought some pencils costing more than a penny each at the school bookstore and paid $\$1.43$. Sharona bought some of the same pencils and paid $\$1.87$. How many more pencils did Sharona buy than Jamar?

2. In a middle-school mentoring program, a number of the sixth graders are paired with a ninth-grade student as a buddy. No ninth grader is assigned more than one sixth-grade buddy. If $\frac{1}{3}$ of all the ninth graders are paired with $\frac{2}{5}$ of all the sixth graders, what fraction of the total number of sixth and ninth graders have a buddy?

3. Let R be a set of nine distinct integers. Six of the elements are 2, 3, 4, 6, 9, and 14. What is the number of possible values of the median of R ?

4. Suppose that $\{a_n\}$ is an arithmetic sequence with $a_1+a_2+\cdots+a_{100}=100 \quad \text{and} \quad a_{101}+a_{102}+\cdots+a_{200}=200.$ What is the value of a_2-a_1 ? Express your answer as a common fraction.

5. Let a and b be distinct real numbers for which $\frac{a}{b}+\frac{a+10b}{b+10a}=2.$ Find $\frac{a}{b}$ in simplest form.

6. Suppose that a parabola has vertex $\left(\frac{1}{4},-\frac{9}{8}\right)$, and equation $y=ax^2+bx+c$, where $a>0$ and $a+b+c$ is an integer. What is the minimum possible value of a? Express your answer as a common fraction.

7. The real root of the equation $8x^3-3x^2-3x-1=0$ can be written in the form $\frac{\sqrt[3]{a}+\sqrt[3]{b}+1}{c}$, where a, b, and c are positive integers. Find $a+b+c$.

8. Find the number of positive integers n less than 1000 for which there exists a positive real number x such that $n=x\lfloor x\rfloor$.

9. Consider the sequence $(a_k)_{k\geq 1}$ of positive rational numbers defined by $a_1=\frac{2020}{2021}$ and for $k\geq 1$, if $a_k=\frac{m}{n}$ for relatively prime positive integers m and n, then $a_{k+1}=\frac{m+18}{n+19}$. Determine the sum of all positive integers j such that the rational number a_j can be written in the form $\frac{t}{t+1}$ for some positive integer t.

Answers:

1. 4
2. 4/11
3. 7
4. 1/100
5. 4/5
6. 2/9
7. 98
8. 496
9. 59