Individual

1. Evaluate $\dfrac{100-99+98-97\cdots+4-3+2-1}{1-2+3-4\cdots+97-98+99-100}$.

2. Ash is running around town catching Pokémon. Each day, he may add $3, 4$, or 5 Pokémon to his collection, but he can never add the same number of Pokémon on two consecutive days. What is the smallest number of days it could take for him to collect exactly 100 Pokémon?

3. Find x, where x is the smallest positive integer such that 2^x leaves a remainder of 1 when divided by 5, 7, and 31.

4. How many positive integer divisors of 201^9 are perfect squares or perfect cubes (or both)?

5. Compute the sum of all even positive integers n less than 1000 such that lcm(1,2,...,n-1) is not equal to lcm(1,2,...n).

6. Suppose that real number x satisfies $\sqrt{49-x^2}-\sqrt{25-x^2}=3.$ What is the value of $\sqrt{49-x^2}+\sqrt{25-x^2}$?

7. Let $P(x)=x^2+4x+1$. What is the product of all real solutions to the equation $P(P(x))=0$? Express your answer in simplest radical form.

8. What is the least possible value of $(x+1)(x+2)(x+3)(x+4)+2019$ where x is a real number?

9. Let n = $2022^{\left(2022^{\cdot^{\cdot^{\left(2022^{2022}\right)}}}\right)}$ where there are 2022 2022 s. What is the remainder when n is divided by 111?

10. Real numbers x and y satisfy

$$x^2+y^2=2023$$
$$(x-2)(y-2)=3.$$

Find the largest possible value of $|x-y|$. Express your answer in simplest radical form.

Team Round

1. We know that
$2x+3y+3z=8,$
$3x+2y+3z=808,$
$3x+3y+2z=80808.$
What is x+y+z?

2. Find $a+b+c$, where $a, b,$ and c are the hundreds, tens, and units digits of the six-digit number $123abc$, which is a multiple of 990.

3. Find the number of lattice points that the line $19x+20y=1909$ passes through in Quadrant I.

4. Find the remainder when $712!$ is divided by 719.

5. Find the largest natural number n such that $2^n+2^{11}+2^8$ is a perfect square.

6. Find the smallest positive integer n for which 315^2-n^2 evenly divides 315^3-n^3.

7. An arithmetic sequence of exactly 10 positive integers has the property that any two elements are relatively prime. Compute the smallest possible sum of the 10 numbers.

8. Consider digits $\underline{A}, \underline{B}, \underline{C}, \underline{D}$, with $\underline{A} \neq 0,$ such that $\underline{ABCD} = (\underline{CD})^2 - (\underline{AB})^2.$ Compute the sum of all distinct possible values of $\underline{A}+\underline{B}+\underline{C}+\underline{D}$.

9. Find the number of ordered triples of positive integers $(a, b, c),$ where $1 \leq a, b, c \leq 10,$ with the property that $\gcd(a, b), \gcd(a, c),$ and $\gcd(b, c)$ are all pairwise relatively prime.

Answers:

Handout:

1. -1
2. 23
3. 60
4. 37
5. 1022
6. 8
7. 3-sqrt(3)
8. 2018
9. 75
10. sqrt(2197)

Team Round:

1. 10203
2. 12
3. 5
4. 718
5. 12
6. 90
7. 1360
8. 21

9. 841