1. How many distinguishable rearrangements of the letters in CONTEST have both the vowels first?
(A) 60 **(B)** 120 **(C)** 240 **(D)** 720 **(E)** 2520

2. All sides of the convex pentagon $ABCDE$ are of equal length, and $\angle A = \angle B = 90°$. What is the degree measure of $\angle E$?
(A) 90 **(B)** 108 **(C)** 120 **(D)** 144 **(E)** 150

3. The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?
(A) 12 **(B)** 17 **(C)** 24 **(D)** 27 **(E)** 36

4. In a room, 2/5 of the people are wearing gloves, and 3/4 of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?
(A) 3 **(B)** 5 **(C)** 8 **(D)** 15 **(E)** 20

5. Suppose $[a\ b]$ denotes the average of a and b, and $\{a\ b\ c\}$ denotes the average of a, b, and c. What is $\{\{1\ 1\ 0\}\ [0\ 1]\ 0\}$?
(A) $\frac{2}{9}$ **(B)** $\frac{5}{18}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{7}{18}$ **(E)** $\frac{2}{3}$

7. In right $\triangle ABC$ with hypotenuse AB, $BC = 5$ and $AC = 12$, circles are drawn, one with center A and radius 12, the other with center B and radius 5. They intersect the hypotenuse at M and N. Then, MN has length: **(A)** 2 **(B)** $\frac{13}{5}$ **(C)** 3 **(D)** 4 **(E)** $\frac{24}{5}$

8. Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t?
(A) $\left(\frac{1}{5}+\frac{1}{7}\right)(t+1)=1$ **(B)** $\left(\frac{1}{5}+\frac{1}{7}\right)t+1=1$ **(C)** $\left(\frac{1}{5}+\frac{1}{7}\right)t=1$
(D) $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1$ **(E)** $(5+7)t=1$

9. Lucky Larry's teacher asked him to substitute numbers for a, b, c, d, and e in the expression $a-(b-(c-(d+e)))$ and evaluate the result. Larry ignored the parentheses but added and subtracted correctly and obtained the correct result by coincedence. The numbers Larry substituted for a, b, c, and d were 1, 2, 3, and 4, respectively. What number did Larry substitute for e?
(A) -5 **(B)** -3 **(C)** 0 **(D)** 3 **(E)** 5

10. For how many positive integers m is it true that there exists a positive integer n such that $mn < m+n$
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** $infinitelymany$

11. Which of the following numbers is a perfect square?
(A) $98! \cdot 99!$ **(B)** $98! \cdot 100!$ **(C)** $99! \cdot 100!$ **(D)** $99! \cdot 101!$ **(E)** $100! \cdot 101!$

12. What is the smallest positive integer that is neither prime nor square and that has no prime factor less than 50?
(A) 3127 **(B)** 3133 **(C)** 3137 **(D)** 3139 **(E)** 3149

13. How many pairs of positive integers (a,b) with $a+b \le 100$ satisfy the equation $\frac{a+b^{-1}}{a^{-1}+b} = 13$?
(A) 1 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 13

14. The letters A, B, C and D represent digits. If $\begin{array}{cc} & A\ B \\ + & C\ A \\ \hline & D\ A \end{array}$ and $\begin{array}{cc} & A\ B \\ - & C\ A \\ \hline & \ \ A \end{array}$, what digit does D represent?
(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

15. For a function $f(x)$, the equation $3f(x)+2f\left(\frac{\sqrt{6}}{6x}\right)=x^2$ for all x except $x=0$. What is $12f(2)$?
(A) $\frac{127}{10}$ **(B)** $\frac{143}{10}$ **(C)** 15 **(D)** $\frac{127}{5}$ **(E)** $\frac{143}{5}$

1 Team Play! from Mandelbrot

Setup: On this Team Play we will investigate certain sequences of positive odd integers. To begin, choose an odd integer M greater than 1, which will be the master number for our sequence. Then write down the number 1, which will be the first term in our sequence. To obtain the next term, subtract the current term from M, record the number of factors of 2 in the result, and then divide out all these factors of 2. Continue this process to build the entire sequence. For example, suppose that we chose M=13 to generate a sequence. As instructed, we let the first term equal 1. To find the next term we compute 13–1=12, note that there are two factors of 2 in 12, and divide them out to obtain 3, the second term. We can indicate this process more compactly by writing 1—>3(2), where the 2 in parentheses means that we divided out two factors of 2. To determine the third term we compute 13–3=10, then divide out a single 2 to obtain 5. Finally, we calculate 13–5=8 and divide out three factors of 2 to obtain 1. At this point the sequence will clearly begin to repeat, so we don't need to proceed further. In summary, for M=13 our sequence looks like 1—>3(2)—>5(1)—>1(3).

Part i: Construct sequences using the same format as in the Setup section for the master numbers M=11, 23, 29, and 37.

Part ii: Now consider the sequence for M=1123. (Do not attempt to construct this sequence!) What number comes just before the number 5 in this sequence?

Part iii: The sequence for M=73 is unusually short: 1—>9(3)—>1(6). Find a value $M > 100$ whose sequence also has exactly two terms before it begins to repeat.

Part iv: For some values of M, such as those in the first part, the resulting sequence contains every positive odd number less than or equal to (M–1)/2. We say that such a sequence is complete. Prove that if M is composite (i.e. not prime) then the corresponding sequence will not be complete.

Part v: In each of the four sequences in the first part, determine the total number of factors of 2 that were divided out before the sequence began repeating. For example, in the Setup section we divided out 2+1+3=6 factors of 2 in all. Make a conjecture relating this total to the value of M used to generate the sequence. Then prove that your conjecture holds for all complete sequences.