Denver Math Club September 2021 Meeting Factoring & Polynomials Handout

- 1. A polynomial written in the form x^2+bx+c has roots of 7 and 9. Find the sum of b and c.
- 2. Find the roots of the polynomial $x^2-9x+20$
- 3. Find the roots of the polynomial $4x^2-98x+360$
- 4. A quadratic equation $ax^2 2ax + b = 0$ has two real solutions. What is the average of these two solutions?
- 5. Let a and b be the roots of the equation $x^2 mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are the roots of the equation $x^2 px + q = 0$. What is q?
- 6. Let f be a function for which $f\left(\frac{x}{3}\right) = x^2 + x + 1$. Find the sum of all values of z for which f(3z) = 7.
- 7. The quadratic equation $x^2 + mx + n$ has roots twice those of $x^2 + px + m$, and none of m, n, and p is zero. What is the value of n/p?

8. What is the sum of the reciprocals of the roots of the equation $\frac{2003}{2004}x + 1 + \frac{1}{x} = 0$?

- 9. The polynomial $x^3 ax^2 + bx 2010$ has three positive integer roots. What is the smallest possible value of a?
- 10. There are two values of a for which the equation $4x^2 + ax + 8x + 9 = 0$ has only one solution for x. What is the sum of these values of a?

Denver Math Club September 2021 Meeting Algebraic Manipulations Handout

1. Let x and y be real numbers such that 2 < (x - y)/(x + y) < 5. If x/y is an integer, what is its value?

2. Find the sum of all positive integers n for which $n^2 - 19n + 99$ is a perfect square.

3. Suppose x > 1 is a real number such that $x + 1x = \sqrt{22}$. What is $x^2 - 1x^2$? Express your answer in simplest radical form.

4. Let $x_1, x_{20}, \ldots, x_{10}$ be 10 numbers. Suppose that $x_i + 2x_{i+1} = 1$ for each i from 1 through 9. What is the value of $x_1 + 512x_{10}$?

5. The parabolas $y = x^2 + 15x + 32$ and $x = y^2 + 49y + 593$ meet at one point (x_0, y_0) . Find $x_0 + y_0$.

6. Suppose x and y are nonzero real numbers simultaneously satisfying the equations x + 2018/y = 1000 and 9/x + y = 1. Find the maximum possible value of x + 1000y.

7. Find the unique real number c such that the polynomial $x^3 + cx + c$ has exactly two real roots.

8. Let a be a real number satisfying the equation equation $(1 + a)^2 + a^2 = 1337$. Then, $(1 + a)^3/(1+a^3) = m/n$, where m and n are positive coprime integers. Find m+n.

9. Suppose a, b, and c are nonzero real numbers such that (bc + 1/a) = (ca + 2/b) = (ab + 7/c) = 1/(a + b + c). Find $(a + b + c)^3$, expressing your answer as a common fraction.

10. Let a and b be the solutions to the equation

 $(x-1)(x-4)(x-2)(x-8)(x-5)(x-7) + 48\sqrt{3} = 0$. Find $(a-b)^2$, expressing your answer in simplest radical form.