Denver Math Club<br>September 2021 Meeting<br>Factoring \& Polynomials Handout

1. A polynomial written in the form $x^{2}+b x+c$ has roots of 7 and 9 . Find the sum of $b$ and $c$.
2. Find the roots of the polynomial $x^{2}-9 x+20$
3. Find the roots of the polynomial $4 x^{2}-98 x+360$
4. A quadratic equation $a x^{2}-2 a x+b=0$ has two real solutions. What is the average of these two solutions?
5. Let $a$ and $b$ be the roots of the equation $x^{2}-m x+2=0$. Suppose that $a+\frac{1}{b}$ and $b+\frac{1}{a}$ are the roots of the equation $x^{2}-p x+q=0$. What is $q$ ?
6. Let $f$ be a function for which $f\left(\frac{x}{3}\right)=x^{2}+x+1$. Find the sum of all values of $z$ for which $f(3 z)=7$.
7. The quadratic equation $x^{2}+m x+n$ has roots twice those of $x^{2}+p x+m$, and none of $m, n$, and $p$ is zero. What is the value of $n / p$ ?
8. What is the sum of the reciprocals of the roots of the equation $\frac{2003}{2004} x+1+\frac{1}{x}=0$ ?
9. The polynomial $x^{3}-a x^{2}+b x-2010$ has three positive integer roots. What is the smallest possible value of $a$ ?
10. There are two values of $a$ for which the equation $4 x^{2}+a x+8 x+9=0$ has only one solution for $x$. What is the sum of these values of $a$ ?

Denver Math Club<br>September 2021 Meeting<br>Algebraic Manipulations Handout

1. Let $x$ and $y$ be real numbers such that $2<(x-y) /(x+y)<5$. If $x / y$ is an integer, what is its value?
2. Find the sum of all positive integers $n$ for which $n^{2}-19 n+99$ is a perfect square.
3. Suppose $x>1$ is a real number such that $x+1 x=\sqrt{22}$. What is $x^{2}-1 x^{2}$ ? 1 Express your answer in simplest radical form.
4. Let $\mathrm{x}_{1}, \mathrm{x}_{20}, \ldots, \mathrm{x}_{10}$ be 10 numbers. Suppose that $\mathrm{x}_{\mathrm{i}}+2 \mathrm{x}_{\mathrm{i}+1}=1$ for each i from 1 through 9 . What is the value of $x_{1}+512 x_{10}$ ?
5. The parabolas $y=x^{2}+15 x+32$ and $x=y^{2}+49 y+593$ meet at one point $\left(x_{0}, y_{0}\right)$. Find $x_{0}+$ уо.
6. Suppose x and y are nonzero real numbers simultaneously satisfying the equations $x+2018 / y=1000$ and $9 / x+y=1$. Find the maximum possible value of $x+1000 y$.
7. Find the unique real number c such that the polynomial $\mathrm{x}^{3}+\mathrm{cx}+\mathrm{c}$ has exactly two real roots.
8. Let a be a real number satisfying the equation equation $(1+a)^{2}+a^{2}=1337$. Then, $(1+a)^{3} /\left(1+a^{3}\right)=m / n$, where $m$ and $n$ are positive coprime integers. Find $m+n$.
9. Suppose $\mathrm{a}, \mathrm{b}$, and c are nonzero real numbers such that $(\mathrm{bc}+1 / \mathrm{a})=(\mathrm{ca}+2 / \mathrm{b})=(\mathrm{ab}+7 / \mathrm{c})=$ $1 /(a+b+c)$. Find $(a+b+c)^{3}$, expressing your answer as a common fraction.
10. Let a and b be the solutions to the equation
$(x-1)(x-4)(x-2)(x-8)(x-5)(x-7)+48 \sqrt{3}=0$. Find $(a-b)^{2}$, expressing your answer in simplest radical form.
