

1. What is the units digit of  $2012^{2013}$  ?

2. What is the sum of the solutions of  $\log_2((x+3)(x+2)) = 7$ ?

3. Let  $ABCD$  be a trapezoid of area 48 with  $\overline{AB}$  parallel to  $\overline{DC}$ ,  $AB = 9$ , and  $DC = 3$ . Diagonals  $\overline{AC}$  and  $\overline{BD}$  intersect in point  $E$ . What is the area of  $\triangle ABE$  ?

4. A coin falls onto a checkerboard. The center of the coin is equally likely to lie on any point of the board. The coin is 0.5 inches in diameter and the lines on the board are 2 inches apart. What is the probability that the coin lies on a side of at least one square?

5. For how many integer values of  $k$  does the equation

$$(x - k)^2 + kx - k^2 = \frac{3}{4} - k$$

have no real solutions for  $x$ ?

6. How many 10-digit integers consist of two 2s, three 3s, and five 5s?

7. How many ordered triples of positive integers  $(a, b, c)$  satisfy

$$a^2 + b^2 + c^2 = 27$$

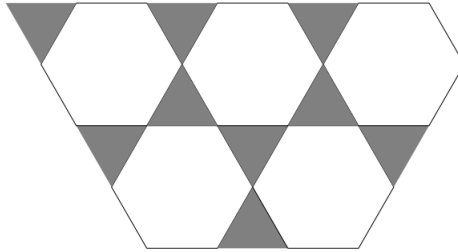
8. Positive integers  $a, b, c$  with  $a < b < c$  form a geometric sequence with an integer ratio. If  $c = 2009$ , what is the value of  $a$ ?

9. A heavy bucket two-thirds full of water is put on a scale and the scale reads  $a$  kg. If the bucket is half full of water, the scale reads  $b$  kg. If the bucket is full of water, what, in terms of  $a$  and  $b$ , would the scale read?

10. If a positive integer factor of 75,600 is chosen at random, what is the probability that it is also a factor of 5,402,250?

11. If  $0 \leq X \leq 1$ , what is the least value that the product  $X(1 - X)$  never exceeds?

12. Shaded triangles and white hexagons cover the plane as shown. What proportion of the plane is shaded?



13. A parking lot has 5 cars with unique license plate numbers. What is the probability that the license numbers of the first 3 cars to leave the parking lot are in increasing order?

14. If a fair coin is tossed 4 times, what is the probability that the number of heads equals the number of tails?

15. If a positive integer factor of 174,636,000 is chosen at random, what is the probability that it is odd?

16. Assume that  $y = ax^2 + bx + a$ . What does  $a$  equal?

17. A pizza restaurant offers a basic pizza with the optional addition of up to 3 of the following extra toppings: anchovies, salami, onions, pepperoni, sausage, mushrooms, roasted peppers, broccoli, and sun-dried tomatoes. Using anywhere from none to at most 3 extra toppings, how many different kinds of pizza are possible?

18. A solitaire game is played  $n$  times with  $x$  wins and  $y$  losses. The percentage of wins to total games is 99%. If the next game is a loss then how many consecutive games must be won with no losses so that the percentage of wins to total games is again 99% ?

19. In how many ways can 180 be written as the sum of a sequence of consecutive odd positive integers?

20. Evaluate

$$\left( \left( \frac{\log_2 16}{\log_4 16} \right)^{-1} \right)^{\log_3 81}$$