

1. What is the units digit of 2012^{2013} ?

2. What is the sum of the solutions of $\log_2((x+3)(x+2)) = 7$?

3. Let $ABCD$ be a trapezoid of area 48 with \overline{AB} parallel to \overline{DC} , $AB = 9$, and $DC = 3$. Diagonals \overline{AC} and \overline{BD} intersect in point E . What is the area of $\triangle ABE$?

4. A coin falls onto a checkerboard. The center of the coin is equally likely to lie on any point of the board. The coin is 0.5 inches in diameter and the lines on the board are 2 inches apart. What is the probability that the coin lies on a side of at least one square?

5. For how many integer values of k does the equation

$$(x - k)^2 + kx - k^2 = \frac{3}{4} - k$$

have no real solutions for x ?

6. How many 10-digit integers consist of two 2s, three 3s, and five 5s?

7. How many ordered triples of positive integers (a, b, c) satisfy

$$a^2 + b^2 + c^2 = 27$$

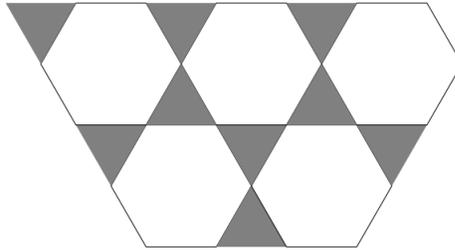
8. Positive integers a, b, c with $a < b < c$ form a geometric sequence with an integer ratio. If $c = 2009$, what is the value of a ?

9. A heavy bucket two-thirds full of water is put on a scale and the scale reads a kg. If the bucket is half full of water, the scale reads b kg. If the bucket is full of water, what, in terms of a and b , would the scale read?

10. If a positive integer factor of 75,600 is chosen at random, what is the probability that it is also a factor of 5,402,250?

11. If $0 \leq X \leq 1$, what is the least value that the product $X(1 - X)$ never exceeds?

12. Shaded triangles and white hexagons cover the plane as shown. What proportion of the plane is shaded?



13. A parking lot has 5 cars with unique license plate numbers. What is the probability that the license numbers of the first 3 cars to leave the parking lot are in increasing order?

14. If a fair coin is tossed 4 times, what is the probability that the number of heads equals the number of tails?

15. If a positive integer factor of 174,636,000 is chosen at random, what is the probability that it is odd?

16. Assume that $y = ax^2 + bx + a$. What does a equal?

17. A pizza restaurant offers a basic pizza with the optional addition of up to 3 of the following extra toppings: anchovies, salami, onions, pepperoni, sausage, mushrooms, roasted peppers, broccoli, and sun-dried tomatoes. Using anywhere from none to at most 3 extra toppings, how many different kinds of pizza are possible?

18. A solitaire game is played n times with x wins and y losses. The percentage of wins to total games is 99%. If the next game is a loss then how many consecutive games must be won with no losses so that the percentage of wins to total games is again 99% ?

19. In how many ways can 180 be written as the sum of a sequence of consecutive odd positive integers?

20. Evaluate

$$\left(\left(\frac{\log_2 16}{\log_4 16} \right)^{-1} \right)^{\log_3 81}$$