Invariants and Monovariants

1. A regular chessboard has two opposite corners removed. Is it possible to tile the remaining 62 squares with 31 dominos?

2. Some people are in a building with several rooms. Each minute a person leaves a room and enters one with at least as many people. Show that eventually all people are in one room.

3. On an $n \times n$ board there are $n^2$ unit squares, $n-1$ of which are infected. Every second, every square sharing an edge with at least two infected squares becomes infected. Show that there will be at least one square which remains uninfected.

4. Peter has 3 accounts in a bank, each with an integral number of dollars. He is only allowed to transfer money from one account to another so that the amount of money in the latter is doubled. Prove that Peter can always transfer all his money into two accounts. Can he always transfer all his money into one account?

5. Numbers 1,2,...,2019 are written in a certain order. Any two neighboring numbers can be replaced by their difference. This operation is repeated until we end up with just one number. Can this number be 0?

6. A group of friends got together to help each other monetarily. A stimulus consists of each friend keeping half of her money and spreading the other half equally among her friends. At the end, all totals are rounded up to the nearest dollar, courtesy of a random benefactor. A series of stimuli terminates if further applications of stimuli no longer change the amounts of money held by alumni. Is there a set of initial dollar distributions such that the series of stimuli never terminates?

7. Nine checkers occupy the 3 by 3 lower left square of a 30 by 30 checkerboard. In a move, you can have one checker jump over another one horizontally, vertically, or diagonally into an empty square. Is there a series of moves which transfers the nine checkers to: A) the lower right 3 by 3 square of the board? B) the upper right 3 by 3 square of the board?